Risk-neutral moments: New theory and evidence under market frictions*

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Abstract

The widely used Bakshi et al. (2003) formulae to estimate risk-neutral moments (RNMs) assume frictionless markets. For the first time, we provide generalised formulae in line with the empirical evidence that market frictions exist in the underlying asset. We relate the estimation of RNMs to market frictions by relying on the concept of the martingale restriction (MR). The violation of MR, that is, the deviation of the expected risk-neutral return from the risk-free rate, implies the presence of market frictions. We find that in this case, the estimation bias in risk-neutral skewness (RNS) is economically significant. The previously documented ability of RNS to predict stock returns arises because its estimation bias, proxies the effect of market frictions on expected stock returns.

JEL classification: G12, G13, G14

Keywords: Martingale restriction, Market frictions, Risk-neutral moments, Risk-neutral skewness, Return predictability

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1 Introduction

The moments of the risk-neutral probability distribution of a specific asset's returns are termed risk-neutral moments (RNMs). RNMs are extracted from the market prices of options written on the asset of interest. They are the building block in the vast literature which examines the informational content of market option prices, to address a number of topics in finance, such as asset pricing, asset allocation, and stock return and economic growth predictability (see Giamouridis and Skiadopoulos (2011), Christoffersen et al. (2013), Bali et al. (2016), Faccini et al. (2019), and references therein). Typically, this literature employs risk-neutral volatility, risk-neutral skewness, and risk-neutral kurtosis, as cases of RNMs. The Bakshi et al. (2003) (BKM) formulae is the current de facto way to estimate these RNMs. Their appeal stems from their model-free nature; they make no assumptions on the risk-neutral distribution, and they only require the market prices of options as inputs. However, the BKM formulae are based on the assumption that the underlying asset market is frictionless. Market frictions, such as short-sale constraints and transaction costs, exist though. This questions the current practice of estimating RNMs. In this paper, we revisit the estimation of RNMs, in the presence of market frictions in the underlying asset. We provide generalised formulae to estimate RNMs, and we show that the estimation bias, due to not taking frictions into account, can be economically significant. To the best of our knowledge, this is the first study which provides generalised Bakshi et al. (2003) (BKM) formulae in a non-frictionless setting.

We develop our approach as follows. First, we recognise that RNMs are related to the risk-neutral expected return of the underlying asset, which we relate to market frictions. To this end, we revisit the *martingale restriction* (MR) under a formal theoretical setting, which takes market frictions into account. MR is a property of asset prices, in the absence of market frictions; if the market is frictionless and arbitrage-free, asset prices discounted by the risk-free rate should be martingales, under the risk-neutral probability measure (first fundamental theorem of asset pricing; Harrison and Kreps, 1979; Harrison and Pliska, 1981). MR predicts that the risk-neutral expected asset return equals the risk-free rate. In the case where MR does not hold, there will be a wedge between the risk-neutral expected asset return and the risk-free rate. A non-zero wedge signifies that MR is violated, due to the presence of market frictions.

Next, we show theoretically how to estimate RNMs in a compatible way with the possible violation of MR in the underlying market. The original BKM formulae (O-RNMs) mis-estimate RNMs, in the case where MR is violated in the underlying market. The bias in the estimation of O-RNMs arises from two sources. The first source is the usage of a wrong risk-neutral expected underlying asset's return to calculate the *central* moments (i.e., moments around the mean) of returns. The original BKM formulae rely on the implicit assumption that the risk-neutral expected asset return equals the risk-free rate. This assumption does not hold though, if MR is violated in the underlying market. We remedy this drawback by providing the generalized BKM formulae to estimate RNMs (G-RNMs). The G-RNM formulae are similar to the BKM formulae, except that the risk-neutral expected underlying asset's return equals the sum of the risk-free rate and the return wedge between the risk-free rate and expected underlying return. A remark is in order. To derive the G-RNM formulae, we acknowledge that we maintain the assumption that option prices satisfy MR. This is equivalent to assuming that the option market is frictionless. Admittedly, this assumption is also violated in practice. However, it is necessary to allow calculating RNMs in a model-free way, as the original BKM formulae do. We prefer taking a "step-by-step" approach, and explore first any effects of the violation of MR in the underlying market, before switching to a non model-free setting, where other strong assumptions will be required (e.g., on modeling market frictions).

The second source of O-RNMs bias stems from using the Black and Scholes (1973) implied volatilities (BS-IV) to compute O-RNMs.[1] Taking risk-neutral skewness (RNS) as an example, we show that in a simulated setting, the use of the standard BS-IV, as an input to interpolate option prices, yields a mis-estimated RNS, in the case where MR is violated in the underlying market. Interestingly, the slope of BS-IV curves (measured as the difference between out-of-the-money and at-the-money option's BS-IV) is frequently interpreted as a proxy of RNS (e.g., Bali et al., 2018). We show that in the presence of market frictions, a non-zero BS-IV slope measure does not necessarily mean that the underlying distribution is not log-normal, in contrast to the common perception. To remedy the drawbacks of BS-IVs, we propose the *robust* IV, which accounts for the possible violation of MR in the calculation of IV. Our simulation results show that the

[1] The implementation of the theoretical formulae of BKM requires a continuum of option prices with respect to the strike price. However, market option prices are available only for discrete strikes. To obtain a continuum of option prices, typically, one interpolates/extrapolates the BS-IVs of the corresponding traded options. Then, she transforms the obtained continuum of BS-IVs to their corresponding option prices.

estimation bias in RNS decreases, once we interpolate option prices in the space of the robust, rather than BS IVs. Moreover, the robust IV ensures that the slope of the robust IV curve satisfies an elementary property: a flat robust IV curve corresponds to log-normally distributed asset returns.

To operationalize empirical exercises, we employ the Spot and Synthetic Difference (SSD) measure proposed by Hiraki and Skiadopoulos (2023) (HS). HS demonstrate that SSD provides a good estimation of the return wedge between the risk-free rate and the expected risk-neutral stock return. Therefore, by using SSD, the calculation of the generalized BKM formulae and the robust IV is feasible. Equipped with SSD, first we test whether the S&P 500 index and U.S. individual equities violate MR, by examining the size of their respective SSD. We find that both the index and individual stocks frequently violate MR; their SSD is economically and statistically significant. The 30-day risk-neutral expected return of the S&P 500 index, deviates from the risk-free rate by 1% to 2% per year, on average. The degree of the violation of MR is greater for individual stocks. The time-series average deviation of the 30-day risk-neutral expected return of individual stocks from the risk-free rate, is greater than 1% per year, for almost all stocks, and greater than 3% for more than half of the stocks. These findings validate our motivation to theoretically and empirically investigate the effect of the violation of MR, and hence of frictions, on the estimated RNMs.

Next, we compare the estimated O-RNMs and G-RNMs of the U.S. individual equities. We find that the violation of MR due to market frictions, affects mostly the estimated RNS, rather than the estimated model-free risk-neutral volatility, and kurtosis. The correlations between O-RNMs and G-RNM, for the case of the latter two RNMs, is almost perfect. Importantly, we find that the difference between O-RNS and G-RNS is economically significant; we find that O-RNS predicts the cross-section of future returns, as already documented by previous literature, whereas G-RNS, which takes market frictions into account, does not. The latter finding is novel, and echoes Hou et al. (2018). They document that the vast majority of asset pricing anomalies cease to exist, when the effect from microcap stocks is mitigated; small size stocks are subject to larger market frictions. Furthermore, we document that the predictive power of O-RNS stems from its "bias" component caused by the violation of MR. The bias component is highly correlated with SSD, which has been found to predict stock returns (Hiraki and Skiadopoulos (2023), HS). This high correlation is not surprising, given that SSD measures the degree of the violation of MR, arising from market frictions. We find that O-RNS predicts future stock returns only for stocks whose SSD is sufficiently different from zero, that is, for stocks whose MR is significantly violated. In a nutshell, O-RNS predicts future returns because its bias component contains predictive power inherited from SSD, previously documented in HS.

In addition to contributing to the literature on the estimation of RNMs, our study contributes further to three strands of literature. First, it contributes to the literature on the relation between asset returns and market frictions. A voluminous literature has documented that market frictions affect asset returns, under the physical probability measure.[2] However, relatively little research has been done on the effect of market frictions to the estimation of option-implied variables, under the risk-neutral probability measure. Previous studies assume that the underlying satisfies MR, and examine the effect of empirical regularities, such as discretely traded strikes and measurement errors, to the estimation of risk-neutral distributions and IVs (Bliss and Panigirtzoglou (2002), and Hentchel (2003), respectively), and RNMs (Dennis and Mayhew (2009), Ammann and Feser (2019)). Instead, we examine, for the first time, a theoretically fundamental type of bias in RNMs, that arising from the violation of MR. Moreover, we show that this bias occurs even under an ideal situation, where a continuum of option prices are observable, in the absence of any measurement errors.

Second, we contribute to the literature on testing MR in stock markets (Longstaff (1995), Strong and Xu, 1999, Neumann and Schlag, 1996, Huang et al., 2016, Guo et al., 2013). These studies test MR by examining the difference between the market stock price, and the implied stock price extracted from an option pricing model (typically the Black and Scholes, 1973, model). They also find that MR is violated. However, we contribute to this literature by proposing a novel, easy to be implemented, and more robust approach to test MR; the test is performed by assessing the size of SSD. Our approach has two desirable properties. First, our test is theoretically founded, based

[2]Early studies by He and Modest (1995) and Luttmer (1996) argue that the equity risk premium puzzle can be explained by considering the effect of market frictions on the expected asset returns. Brunnermeier and Pedersen (2009), Gârleanu and Pedersen (2011), Chabakauri (2013), and Frazzini and Pedersen (2014), among others, show that margins and leverage constraints affect asset returns. A number of studies document that short-sale constraints affect stock returns (e.g., Chen et al., 2002; Ofek et al., 2004; Asquith et al., 2005; Drechsler and Drechsler, 2014). Moreover, Hou et al. (2018) examine more than 100 market friction-related stock anomaly variables.

on a reformulation of the MR restriction, in the case where frictions are present. We demonstrate that the implied stock price approach may reject MR spuriously, even if MR holds. For example, we show that the BS-model-based implied stock price approach does not constitute a valid MR test, in the case where the true risk-neutral underlying distribution is not log-normal. Previous empirical evidence, based on the implied stock approach, may not be interpreted as violation of MR, but rather as a reflection of the empirical stylized fact that the IVs of out-of-the money (OTM) equity put options are greater than those of at-the-money (ATM) options (negative IV skew). Second, our SSD-based MR test approach is model-free because the estimation of SSD is model-free; HS show that SSD can be reliably estimated in a model-free manner from properly scaled deviations from put-call parity. This is in contrast to the implied stock approach, whose outcome may depend on the choice of the option pricing model.

Finally, our paper is related to the literature which documents that O-RNS positively predicts future stock returns. There is no consensus regarding the mechanism which explains this positive relation, and we contribute to this discussion.[3] The positive relation is explained, either by limits-of-arbitrage (Rehman and Vilkov (2012), Stilger et al. (2017), Gkionis et al. (2018)), or informed option trading (Bali et al. (2018), Borochin et al. (2018)), or by a combination of these two explanations (Chordia et al. (2019)).[4] Our finding that RNS does not predict stock returns, once we account for market frictions, supports the limits-of-arbitrage explanation. In addition, we find that the predictive power of O-RNS is greater among stocks with zero option trading volume, than among stocks with positive option trading volume. This finding is again at odds with the informed option trading story; this predicts that the predictive power of option-implied variables

[3]There is some evidence that that the positive relation between O-RNS and future stock returns may not be robust to choices of the frequency of data, and options' time-to-maturity. Conrad et al. (2013) document that RNS negatively predicts future returns. Borochin et al. (2018) find that there is a positive (negative) relation between O-RNS and future returns, when O-RNS is estimated from short-term (long-term) option data. Our paper does not take a view on this debate; it rather identifies a relationship and examines how this may be affected by the presence of frictions.

[4]The limits-of-arbitrage explanation argues that stocks with more negative O-RNS are relatively overpriced, and this overpricing is not corrected fast due to limits-of-arbitrage. The informed option trading explanation argues that option markets attract informed traders (e.g., Easley et al., 1998). Therefore, RNS reflects their information prior to its revelation in the underlying stock price. For example, pessimistic informed traders may utilize their private information through the option market, by buying OTM put options and selling OTM call options. This trading pressure would increase (decrease) put (call) option prices (Bollen and Whaley, 2004; Gârleanu et al., 2009), resulting in a lower RNS. These two explanations are not necessarily mutually exclusive; the limits-of-arbitrage for trading the underlying, is a prerequisite for option prices to reflect the private information of informed traders (Easley et al. (1998)).

is more pronounced, when calculated from options with a higher trading volume. On the other hand, this finding is consistent with the limits-of-arbitrage explanation. Stocks with zero option trading volume, tend to be smaller stocks, which face a considerable degree of market frictions. Therefore, the violation of MR tends to be severer for them, and as a result, the predictive power of their O-RNS is greater because SSD is greater. This finding echoes Goncalves-Pinto et al. (2017), who show that their option-based predictor of stocks returns, termed DOTS, does not exhibit a stronger predictability among stocks with non-zero option trading volume, versus stocks with zero option trading volume. Note that our SSD explanation for the predictive power of O-RNS can accommodate both positive and negative informational content of O-RNS. This is in contrast with a predictive mechanism based on short-sale constraints, as it can only explain negative informational content of O-RNS only.

The remaining part of the paper is organized as follows. In Section 2, we outlay the theoretical framework by revisiting MR. In Section 3, we derive the generalized BKM formulae to estimate RNMs, and we present the robust IV concept, both of which address the possible violation of MR. Section 4 explains the data sources, provides empirical evidence on the violation of MR, and compares O-RNMs with G-RNMs. Section 5 investigates the return predictive power of O-RNS and G-RNS for future stock returns. Section 6 concludes.

2 Theoretical framework: The martingale restriction

We consider a financial market, where three types of assets trade: the risk-free bond, a risky asset (the stock), and options written on the stock. We assume that the instantaneous risk-free rate r_f is constant. The gross risk-free bond retrun from time t to T is denoted as $R^f_{t,T} = e^{r_f(T-t)}$. Let S_t be the stock price at time t. The stock pays dividends, and $\widetilde{D}_{t,T}$ denotes the time T value of the cumulative dividends paid over the period $(t, T]$,

$$\widetilde{D}_{t,T} = \sum_{t<j\leq T} R^f_{j,T} D_j. \tag{1}$$

We define the cum-dividend gross stock return from t to T by $R_{t,T} = (S_T + \widetilde{D}_{t,T})/S_t$. The time-$t$ price of a call (put) option with strike price K and maturity T is denoted by $C_t(K, T)$ $(P_t(K, T))$, and $\tau = T - t$ denotes the length of return horizon and options' time-to-maturity. We call *discounted (cum-dividend) price process*, the discounted value of the asset with the dividends reinvested (if applicable). For example, the discounted cum-dividend stock price process is defined as $\{(S_t + \widetilde{D}_{0,t})/R^f_{0,t}\}_{t\geq 0}$, whereas the discounted price process of a non-dividend paying asset such as a call option would be $\{C_t(K,T)/R^f_{0,t}\}_{t\geq 0}$.

Definition 2.1 (Martingale restriction)**.** *The martingale restriction (MR) is said to be satisfied if the discounted (cum-dividend) price process of any traded asset is a martingale under a risk-neutral probability measure.*

The first fundamental theorem of asset pricing (FFTAP, Harrison and Kreps, 1979; Harrison and Pliska, 1981) states that in a frictionless market, the no-arbitrage condition is equivalent to the existence of a *risk-neutral probability measure* $\mathbb{Q}$, under which MR is satisfied. On the other hand, in the presence of market frictions, asset prices do not necessarily satisfy MR.

MR yields a number of important implications regarding the price and the expected return of any traded asset. First, under MR, the martingale property of the discounted cum-dividend stock price yields

$$\frac{S_t + \widetilde{D}_{0,t}}{R^f_{0,t}} = \mathbb{E}^{\mathbb{Q}}_t\left[\frac{S_T + \widetilde{D}_{0,T}}{R^f_{0,T}}\right] \quad \Leftrightarrow \quad S_t = \frac{1}{R^f_{t,T}}\mathbb{E}^{\mathbb{Q}}_t[S_T + \widetilde{D}_{t,T}], \tag{2}$$

where $\mathbb{E}^{\mathbb{Q}}_t[.]$ denotes the expectation under the $\mathbb{Q}$-measure, conditional on the information set at time t. Equation (2) is the so-called risk-neutral valuation formula, that is, the current stock price equals the discounted value of the expected future cum-dividend stock price under $\mathbb{Q}$. Second, equation (2) implies that the expected stock return $R_{t,T}$ satisfies

$$\mathbb{E}^{\mathbb{Q}}_t[R_{t,T}] = R^f_{t,T}, \tag{3}$$

that is, the $\mathbb{Q}$-expected stock return equals the risk-free rate.

Third, in the asset pricing literature, FFTAP is stated under the physical measure $\mathbb{P}$ (e.g., Cochrane, 2005). For each risk-neutral measure, a corresponding stochastic discount factor (SDF) is defined as $m_{t,T} = e^{-r_f\tau}(d\mathbb{Q}/d\mathbb{P})$, where $d\mathbb{Q}/d\mathbb{P}$ is the Radon-Nikodým derivative for the change of measure from $\mathbb{P}$ to $\mathbb{Q}$. Then, by definition, $\mathbb{Q}$ and $m_{t,T}$ are related through the following change of measure equation,

$$\mathbb{E}_t^{\mathbb{P}}[m_{t,T}(S_T + \widetilde{D}_{t,T})] = \frac{1}{R_{t,T}^f}\mathbb{E}_t^{\mathbb{Q}}[S_T + \widetilde{D}_{t,T}]. \tag{4}$$

Equations (2) and (4) yield

$$1 = \mathbb{E}_t^{\mathbb{P}}[m_{t,T}R_{t,T}]. \tag{5}$$

Moreover, equation (5) yields the following asset pricing equation:

$$\mathbb{E}_t^{\mathbb{P}}[R_{t,T}] - R_{t,T}^f = -R_{t,T}^f Cov_t^{\mathbb{P}}(m_{t,T}, R_{t,T}), \tag{6}$$

that is, the expected excess return is determined by the covariance between the SDF and the asset's return.

Therefore, testing MR is of paramount importance because equations (2) to (6), which are the cornerstones of asset pricing, hold, if and only if MR holds. Specifically, we will document that the violation of equation (3) results in biases in the estimated RNMs based on BKM formulae.

The literature suggests that MR is violated in general. To see this point, note that equation (6) indicates that there are no alphas if MR holds. As is well-known, the voluminous literature suggests that alphas or various anomalies exist. Given this literature, we assume that MR does not hold in general and there is a non-zero wedge $\omega_{t,T}$ between the risk-free rate and the $\mathbb{Q}$-expected return (i.e., the violation of equation (3)):

$$\mathbb{E}_t^{\mathbb{Q}}[R_{t,T}] = R_{t,T}^f + \omega_{t,T}. \tag{7}$$

In addition, HS propose a methodology to estimate $\omega_{t,T}$ from option prices, which we we employ in our subsequent empirical analysis.

Relatedly, Longstaff (1995) proposes a way to directly examine whether equation (2) holds and he claims that MR is violated for US equity indices based on his empirical investigations. Note, however, that his proposed approach has a flaw as we discuss in Appendix **X**.

3 Estimation of risk-neutral moments: Theory

In this Section, we develop the theory to estimate RNMs, under the possible violation of MR. In Section 3.1, we provide the *generalized* Bakshi et al. (2003) (BKM) formulae for the estimation of RNMs, in the case where the underlying asset violates MR. Then, in Section 3.2, we investigate the behavior of BS-IV under the violation of MR. This is of importance for studying the effect of violations of MR to RNMs for two reasons. First, IV is an essential ingredient to estimate RNMs; our simulation result in Section 3.3 shows that the usage of the original BKM formulae and the standard IV, results in biases in the estimated RNS, in the case where MR is violated. Second, the slope of IV curves (e.g., the difference between OTM option's IV and ATM option's IV) is frequently interpreted as a proxy of RNS (e.g., Bali et al., 2018). However, in Section 3.4, we show that a non-zero IV slope measure does not necessarily mean that the underlying distribution is not log-normal, in the case where MR is violated. This is in contrast to the common perception that the presence of IV skew indicates departure from log-normality. To remedy these issues, we propose the *robust* IV, which accounts for the possible violation of MR in the calculation of IV.

3.1 The generalized Bakshi et al. (2003) formulae

BKM assume that both the underlying asset and option prices satisfy MR, to derive the original BKM formulae. We relax the assumption that the underlying satisfies MR, whereas we keep the MR assumption for the option prices. Admittedly, the assumption that option prices satisfy MR is strong because it implies that option prices are not affected by market frictions. However, it is not possible to estimate RNMs in a model-free manner, once this assumption is also relaxed. Our formulae also generalize BKM in that we allow the underlying asset to pay dividends.

Let $r_{t,T} = \log((S_T + \widetilde{D}_{t,T})/S_t)$ be the *log cum-dividend return* of the underlying stock. The (generalized) BKM formulae estimate the central moments of the distribution of $r_{t,T}$. The model-free implied volatility (MFIV), risk-neutral skewness (RNS), and risk-neutral kurtosis (RNK) of the $\mathbb{Q}^*$-distribution of $r_{t,T}$ are denoted by $MFIV_{t,T}$, $RNS_{t,T}$, and $RNK_{t,T}$, respectively. We consider a derivative which delivers a payoff $(r_{t,T})^n$ with a positive integer n at time T. We call this derivative the "power-n log return contract", and we denote its present value as $M(n)_{t,T} = e^{-r_f\tau}\mathbb{E}_t^{\mathbb{Q}^*}[(r_{t,T})^n]$.[5]

Proposition 3.1 (Generalized BKM formulae)**.** *Assume that the underlying may violate MR, yet option prices satisfy MR. Then, the MFIV, RNS and RNK are given by*

$$MFIV_{t,T} = \sqrt{\frac{e^{r_f\tau}M(2)_{t,T} - \widetilde{\mu}_{t,T}^2}{\tau}}, \tag{8}$$

$$RNS_{t,T} = \frac{e^{r_f\tau}M(3)_{t,T} - 3e^{r_f\tau}\widetilde{\mu}_{t,T}M(2)_{t,T} + 2\widetilde{\mu}_{t,T}^3}{[e^{r_f\tau}M(2)_{t,T} - \widetilde{\mu}_{t,T}^2]^{3/2}}, \tag{9}$$

$$RNK_{t,T} = \frac{e^{r_f\tau}M(4)_{t,T} - 4e^{r_f\tau}\widetilde{\mu}_{t,T}M(3)_{t,T} + 6e^{r_f\tau}\widetilde{\mu}_{t,T}^2M(2)_{t,T} - 3\widetilde{\mu}_{t,T}^4}{[e^{r_f\tau}M(2)_{t,T} - \widetilde{\mu}_{t,T}^2]^2}, \tag{10}$$

where

$$\widetilde{\mu}_{t,T} = \omega_{t,T} + e^{r_f\tau} - 1 - \frac{e^{r_f\tau}}{2}M(2)_{t,T} - \frac{e^{r_f\tau}}{6}M(3)_{t,T} - \frac{e^{r_f\tau}}{24}M(4)_{t,T} \tag{11}$$

is the forth-order Taylor series approximation of the risk-neutral expected log stock return (i.e., $\widetilde{\mu}_{t,T} \approx \mathbb{E}_t^{\mathbb{Q}}[r_{t,T}]$). The value of the power-n log return contracts $M(n)_{t,T}$ is given by

$$M(n)_{t,T} = \int_{S_t-\widetilde{D}_{t,T}}^{\infty} \eta(K; S_t, n)C_t(K,T)dK + \int_0^{S_t-\widetilde{D}_{t,T}} \eta(K; S_t, n)P_t(K,T)dK, \tag{12}$$

where

$$\eta(K; S_t, \widetilde{D}_{t,T}, n) = \frac{n}{(K+\widetilde{D}_{t,T})^2}\left[(n-1)\log\left(\frac{K+\widetilde{D}_{t,T}}{S_t}\right)^{n-2} - \log\left(\frac{K+\widetilde{D}_{t,T}}{S_t}\right)^{n-1}\right]. \tag{13}$$

Proof. See Appendix A.1. □

Proposition 3.1 provides the formulae to calculate RNMs in a setting where MR may be violated. These formulae modify the BKM formulae in two ways. First, the risk-neutral mean of the log stock return $\widetilde{\mu}_{t,T}$ is modified (equation (11)). The expression for the risk-neutral mean log stock return in BKM's original study (equation (39) of BKM) does not contain the MR violation wedge term $\omega_{t,T}$, reflecting their assumption that the underlying satisfies MR. Therefore, the original BKM formulae mis-estimate the RNMs of the log stock return if MR is violated, because they use a mis-measured mean of the log stock return to calculate central moments.

Second, our formulae consider the cum-dividends return, whereas BKM assume that the underlying pays no dividends. The inclusion of dividends modifies the new formulae

[5] BKM call the power-2, -3, and -4 log return contracts the volatility, cubic and quartic contracts. They also denote $M(n)_{t,T}$ ($n = 2, 3, 4$) by $V_{t,T}$, $W_{t,T}$, $X_{t,T}$, respectively.

in two ways compared to BKM. First, the boundary between the call and put integration regions in equation (12) changes to $S_t - \widetilde{D}_{t,T}$ from S_t in the original BKM formulae. Second, the "weighting" function, equation (13), also changes. These two changes occur because the payoff function changes from the power of the ex-dividend log return $(\log(S_T/S_t))^n$ to that of the cum-dividend log return $(\log((S_T + \widetilde{D}_{t,T})/S_t))^n$.

Proposition 3.1 has provided formulae to estimate the RNMs of the cum-dividend return. The next Proposition, provides formulae for the RNMs of the *ex-dividend return*, $r^{ex}_{t,T} = \log(S_T/S_t)$.

Proposition 3.2 (Generalized BKM formulae for the ex-dividend return)**.** *To estimate the RNMs of the ex-dividend return, the value of power-n contracts is given by*

$$M(n)_{t,T} = \int_{S_t}^{\infty} \eta^{ex}(K; S_t, n) C_t(K,T) dK + \int_0^{S_t} \eta^{ex}(K; S_t, n) P_t(K,T) dK, \tag{14}$$

where

$$\eta^{ex}(K; S_t, n) = \frac{n}{K^2}\left[(n-1)\log\left(\frac{K}{S_t}\right)^{n-2} - \log\left(\frac{K}{S_t}\right)^{n-1}\right]. \tag{15}$$

Moreover, the expected log stock return, equation (11), should be modified to the following equation:

$$\widetilde{\mu}_{t,T} = \omega_{t,T} + e^{r_f\tau} - \frac{\widetilde{D}_{t,T}}{S_t} - 1 - \frac{e^{r_f\tau}}{2}M(2)_{t,T} - \frac{e^{r_f\tau}}{6}M(3)_{t,T} - \frac{e^{r_f\tau}}{24}M(4)_{t,T}. \tag{16}$$

The functional forms of equations (8) to (10) do not change.

Proof. See Appendix A.2. □

Note that equations (14) and (15) are the same as the original BKM formulae. This is because both our formulae in Proposition 3.2 and BKM's original formulae are about the payoff function $\log(S_T/S_t)^n$. However, in contrast to BKM, we assume that the underlying asset pays dividends, and this modifies the mean of the log stock return, equation (16), which now contains an additional term, $-\widetilde{D}_{t,T}/S_t$, reflecting the fact that the ex-dividend expected return is lower than the cum-dividend expected return by $\widetilde{D}_{t,T}/S_t$.

3.2 Violation of MR: Black-Scholes IV and robust IV

Next, we investigate the behavior of BS-IV under the violation of MR. This is of importance because the previous literature typically interpolates and extrapolates Black and

Scholes (1973) implied volatilities (BS-IVs) to obtain a continuum of option prices to implement the BKM formulae and calculate RNMs. To provide an analytically tractable discussion, we consider a situation where the underlying stock may violate MR, yet it follows a log-normal distribution. We assume that the future stock price S_T follows a log-normal distribution under the $\mathbb{Q}$-measure, that is,

$$S_T \sim \text{Lognormal}\left(\log(S_t - e^{-r_S\tau}\widetilde{D}_{t,T}) + r_S\tau - \frac{\sigma^2\tau}{2}, \sigma^2\tau\right), \tag{17}$$

where r_S is the continuously compounded risk-neutral expected return of the stock, $e^{r_S\tau} = \mathbb{E}_t^{\mathbb{Q}}[R_{t,T}]$, and σ is the constant volatility (i.e., annualized standard deviation) parameter.[6]

In the presence of market frictions, MR does not hold, and r_S will differ from r_f:

$$R^f_{t,T} + \omega_{t,T} = \mathbb{E}_t^{\mathbb{Q}}[R_{t,T}] = e^{r_S\tau} \neq e^{r_f\tau} = R^f_{t,T}, \tag{18}$$

where the first equality comes from equation (7).

Next, we calculate the price of options under this setting. To this end, we assume that option prices satisfy MR, and thus option prices are given by

$$C_t(K,T) = e^{-r_f\tau}\mathbb{E}_t^{\mathbb{Q}}[(S_T - K)^+], \quad \textit{and} \quad P_t(K,T) = e^{-r_f\tau}\mathbb{E}_t^{\mathbb{Q}}[(K - S_T)^+], \tag{19}$$

where $(x)^+ = \max(x, 0)$. Then, the call option price is given by

$$C_t(K,T) = e^{(r_S - r_f)\tau} \times e^{-r_S\tau}\mathbb{E}_t^{\mathbb{Q}}[(S_T - K)^+] = e^{(r_S - r_f)\tau} BS_{call}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, \sigma), \tag{20}$$

where $BS_{call}(S, K, \tau, r, D, \sigma)$ is the BS option pricing function with deterministic dividends. The relation $e^{-r_S\tau}\mathbb{E}_t^{\mathbb{Q}}[(S_T - K)^+] = BS_{call}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, \sigma)$ holds because the left-hand side is the price of a call, in the case where the "risk-free rate" is r_S. Similarly, the put option price is given by

$$P_t(K,T) = e^{(r_S - r_f)\tau} BS_{put}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, \sigma). \tag{21}$$

A remark is in order regarding equations (20) and (21). Under the violation of MR, the option pricing functions are a function of seven arguments ($S_t, K, \tau, \sigma, r_f, \widetilde{D}_{t,T}$ and

[6] To show that r_S is the continuously compounded risk-neutral expected return, we calculate the mean of the log-normal distribution, equation (17). This yields

$$\mathbb{E}_t^{\mathbb{Q}}[S_T] = \exp\left(\log(S_t - e^{-r_S\tau}\widetilde{D}_{t,T}) + r_S\tau\right) = e^{r_S\tau}(S_t - e^{-r_S\tau}\widetilde{D}_{t,T}) = e^{r_S\tau}S_t - \widetilde{D}_{t,T}.$$

Some more algebra shows that $\mathbb{E}_t^{\mathbb{Q}}[R_{t,T}] = e^{r_S\tau}$ under the deterministic dividend $\widetilde{D}_{t,T}$ assumption.

r_S), rather than a function of six arguments, as in the case of the BS model. This is because the risk-free rate, used to discount the future expected payoff, differs from the continuously compounded expected stock return, when MR does not hold. The pricing formulae boil down to the standard BS functions with deterministic dividends, in the case where MR is satisfied (i.e., $r_S = r_f$).

The BS-IV is defined implicitly via the following equations:

$$IV_t^c = BS_{call}^{-1}(C_t; S_t, K, \tau, r_f, \widetilde{D}_{t,T}), \quad \textit{and} \quad IV_t^p = BS_{put}^{-1}(P_t; S_t, K, \tau, r_f, \widetilde{D}_{t,T}), \tag{22}$$

where $BS_{call}^{-1}(C; S, K, \tau, r, \widetilde{D})$ ($BS_{put}^{-1}(P; S, K, \tau, r, \widetilde{D})$) is the inverse function of the BS call (put) option function, viewed as a correspondence between the volatility parameter and option price, given other parameters. The following Proposition shows the properties of the BS-IV in the case where the call and put option prices are given by equations (20) and (21), respectively.[7]

Proposition 3.3. *(a) Let $IV_t^c(K)$ ($IV_t^p(K)$) be the BS-IV of the call (put) option price given by equation (20) (equation (21)), that is,*

$$C_t(K,T) = e^{(r_S - r_f)\tau} BS_{call}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, \sigma) = BS_{call}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, IV_t^c(K)) \tag{23}$$

$$P_t(K,T) = e^{(r_S - r_f)\tau} BS_{put}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, \sigma) = BS_{put}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, IV_t^p(K)), \tag{24}$$

Then, $IV_t^c(K)$ and $IV_t^p(K)$ satisfy the following approximate equations, respectively:

$$IV_t^c(K) \approx \sigma + \frac{\omega_{t,T}}{\sqrt{\tau} R_{t,T}^f} \frac{S_t}{S_t - e^{-r_f \tau} \widetilde{D}_{t,T}} \frac{\Phi(d_1)}{\phi(d_1)}, \tag{25}$$

$$IV_t^p(K) \approx \sigma - \frac{\omega_{t,T}}{\sqrt{\tau} R_{t,T}^f} \frac{S_t}{S_t - e^{-r_f \tau} \widetilde{D}_{t,T}} \frac{\Phi(-d_1)}{\phi(-d_1)}, \tag{26}$$

where $\phi(x)$ and $\Phi(x)$ are the probability density function and the cumulative density function of the standard normal distribution and

$$d_1 = \frac{\log \frac{S_t - e^{-r_f \tau} \widetilde{D}_{t,T}}{K} + (r_f + \frac{\sigma^2}{2})\tau}{\sigma \sqrt{\tau}}. \tag{27}$$

(b) When $\omega_{t,T} = 0$, then $IV_t^c(K) = IV_t^p(K) = \sigma$ holds. On the other hand, when

[7] We define BS-IV via the BS model, which prevails in the case of frictionless markets, yet we use a setting with frictions in the underlying asset to show the effect of frictions on BS-IV. Our approach is not internally inconsistent. The fact that we use BS-IV does not mean that market option prices, and the underlying asset price are not affected by frictions; the BS formula serves as a translation mechanism between market option prices and IV, and it does not imply that the BS formula is the correct option pricing formula.

$\omega_{t,T}$ is positive (negative), $IV_t^c(K)$ is greater (smaller) than σ for any strike and decreasing (increasing) in K, whereas $IV_t^p(K)$ is smaller (greater) than σ for any strike and increasing (decreasing) in K.

Proof. See Appendix A.3. □

Proposition 3.3 showcases two interesting findings about the properties of the BS-IV. First, in the case where the underlying violates MR, the BS-IV does not correctly estimate the volatility parameter of the log-normal distribution.[8] For instance, when $\omega_{t,T}$ is positive, the call (put) BS-IVs overestimate (underestimate) the true σ. Figure 1 manifests this finding by showing the IVs given by equations (25) and (26), as a function of moneyness (strike to stock price ratio). We set $\tau = 1/8$ (about 45 days), $r_f = 3\%$, the annualized return wedge $\omega_{t,T}/\tau = 1\%$, the dividend-to-stock $\widetilde{D}_{t,T}/S_t = 0.5\%$ (i.e., 4% annualized dividend yield), and $\sigma = 20\%$.[9] This pattern appears because call (put) options are more expensive (cheaper) compared to the BS model case (i.e., $r_S = r_f$), when $\omega_{t,T} > 0$; a higher stock drift r_S makes call (put) options more (less) likely to expire in-the-money.

[Figure 1 about here.]

Second, in the case where MR is violated, the BS-IVs are not constant across strikes, despite the fact that the risk-neutral distribution of the underlying is assumed to be log-normal. For instance, in the case where $\omega_{t,T}$ is positive, Proposition 3.3 (b) shows that the call (put) IV curve is decreasing (increasing) in K. In this case, the IV skew is a result of market frictions, and it would spuriously suggest that the underlying distribution is not log-normal. This finding is related to Leland (1985) and Çetin et al. (2006), who also show that market frictions may result in skewed IV curve, even if the underlying follows a log-normal distribution.

To remedy the above two drawbacks of the conventional BS-IV (termed *standard* BS-IV, hereafter), we propose the *robust IV*.

[8] Hentchel (2003) shows that the IV can be a biased estimate of the true volatility, due to measurement errors in option prices, the underlying prices, the risk-free rate (which equals the expected stock return under the MR assumption), and other parameters. Our result is related to his findings. In our case, the bias can be interpreted as a result of using the wrong value for the expected stock return, when inverting the BS function.

[9] Note that the numerical value of the wedge variable $\omega_{t,T}$ is in line with the evidence we will present in Section 4, which shows that the wedge of the S&P 500 may be as extreme as ±1%.

Definition 3.1 (Robust IV). *Let r_S be the continuously compounded $\mathbb{Q}$-expected stock return, which differs from r_f, if market frictions are present. Then, the robust IV, IV_{rob}, is defined implicitly via*

$$\begin{aligned} e^{-(r_S-r_f)\tau}C_t(K,T) &= BS_{call}(S_t,K,\tau,r_S,\widetilde{D}_{t,T},IV^c_{rob}), \\ e^{-(r_S-r_f)\tau}P_t(K,T) &= BS_{put}(S_t,K,\tau,r_S,\widetilde{D}_{t,T},IV^p_{rob}). \end{aligned} \tag{28}$$

We implicitly define the robust IV by equating the option price, *adjusted* by $e^{-(r_S-r_f)\tau}$, with the BS model option price, which uses r_S rather than r_f as an input. In contrast, the standard BS-IV is obtained by equating the *unadjusted* option price with the BS model price, which uses the r_f as an input. In the case where the underlying satisfies MR (i.e., $r_S = r_f$), equation (28) boils down to the definition of the standard BS-IV. This suggests that the robust IV generalizes the standard BS-IV by accounting for the possible violation of MR.

The generalization of the standard IV to the robust IV is of importance because the latter eliminates the two drawbacks of the standard IV discussed above, which arise from violations of MR. First, the robust IV correctly estimates the volatility parameter σ, when the underlying distribution is log-normal. This can be shown by comparing equations (20), (21), and (28). Second, in the case where the underlying follows a log-normal distribution, the robust IV is constant across strikes and equals σ, even when MR is violated.

3.3 The effect of market frictions to RNMs: Simulation results

The violation of MR may affect the estimation of RNMs via two channels. The first channel is the theoretical formulae used to compute them. The second channel is the IVs used to obtain a continuum of option prices by interpolation/extrapolation, given that option prices trade for discrete strikes. We conduct a simulation exercise to quantify the effect of each channel to the estimation of RNMs.

For simplicity, we assume that the stock pays no dividends and the stock price follows a log-normal distribution, equation (17). Therefore, the true MFIV, RNS, RNK of the log stock return $\log(S_T/S_t)$ are $\sigma = 30\%$, zero, and three, respectively. We assume that the stock may violate MR (i.e., $r_S \neq r_f$).[10] We set the current stock price $S_t = 100$,

[10] This setup is similar in spirit to Dennis and Mayhew (2009), who analyze how microstructural biases

the risk-free rate $r_f = 3\%$, the volatility $\sigma = 30\%$, and the time-to-maturity $\tau = 1/12$. We consider three cases of the expected stock return r_S: $r_S = r_f$, $r_S = r_f + 3\%$, and $r_S = r_f - 3\%$.

We generate a set of option prices as follows. We assume that options trade at $K = 94, 95, \ldots, 108$. These strikes approximately cover the delta range from 0.2 to 0.8, corresponding to the delta range in the OptionMetrics Volatility Surface file, which we use in the subsequent empirical analysis. We calculate option prices, based on equations (20), and (21). Then, we calculate the standard BS-IV of these options. Unless $r_S = r_f$, the standard BS-IV curve is not flat (Proposition 3.3). On the other hand, the robust IV curve is flat at the level of $\sigma = 30\%$.

In line with the standard estimation procedures in the literature (e.g., Stilger et al., 2017), we interpolate the discrete option IVs by the cubic Hermite polynomial in the moneyness-IV metric, and extrapolate IVs horizontally beyond the highest and lowest strikes. We follow Stilger et al. (2017), and interpolate and extrapolate call and put option IVs, separately. Then, we convert the interpolated IV curves to 1001 option prices over the equally-spaced strikes in the moneyness range $[1/3, 3]$. Finally, we numerically calculate the integrals in the BKM formulae, and calculate the RNMs. To implement these steps, we have two choices regarding the type of IV to be interpolated, and two choices regarding the type of the RNM formulae. These yield four estimation specifications, OS-, OR-, GS-, and GR-RNMs. The first letter of the prefix stands for the type of the formulae (the "Original" BKM formulae or the "Generalized" BKM formulae) and the second letter stands for the type of the IVs used for the IV interpolation (the "Standard" IVs or the "Robust" IVs).

Table 1 reports the result of the estimated RNMs for the four specifications. We can see that the BKM formulae estimate MFIV and RNK accurately, even when MR is violated. On the other hand, they mis-estimate RNS, when MR is violated. The estimation error decreases, as we switch from the BKM to our generalised RNS formula. Moreover, it becomes zero, when we also use the robust IV to compute option prices. These results show that to estimate RNS accurately, one needs to modify the BKM formulae by taking the violation of MR into account, and also use the robust IV for interpolation, and extrapolation purposes.

affect the estimation of RNS, under the log-normality assumption.

[Table 1 about here.]

Next, we explain how the use of BS-IVs contributes to the mis-estimation of RNS, in the presence of market frictions. The extrapolation of the standard BS-IV biases the extrapolated option prices, and this bias transmits to the power-n log return contract. To fix ideas, we consider the case of a positive wedge, as an example. We employ Proposition 3.3, to simulate the call and put standard IV curves over a continuum of strike prices; we treat them as the true, yet unobserved, option prices. Figure 2 depicts the two respective "true" curves (dotted lines), respectively. Then, we treat the simulated IVs for *discrete strikes* $K = 94, 95, \ldots, 108$, as the observed IVs, and we interpolate (extrapolate horizontally) across them (beyond the traded strikes); Figure 2 shows the two respective "observed" curves (solid lines). We can see that the extrapolated call (put) IV line is above (below) the true standard IV line. This shows that the extrapolation, in the space of the standard IVs, overestimates (underestimates) the call (put) option prices if the wedge is positive. The results are revered in the case where the wedge is negative.

[Figure 2 about here.]

The following Lemma examines the sign of the weighting function $\eta(K; S_t, n)$, in equation (13). This helps to explain how the overestimation (underestimation) of OTM call (put) options transmits to the price of the power-n log return contract.

Lemma 3.1. *For odd-degree power-n log return contracts, $\eta(K; S_t, n)$ satisfies*

$$\eta(K; S_t, n) \begin{cases} < 0 & K/S_t < 1, \\ \geq 0 & 1 \leq K/S_t \leq e^{n-1}, \\ < 0 & K/S_t > e^{n-1}. \end{cases} \tag{29}$$

For even-degree power-n log return contracts, $\eta(K; S_t, n)$, satisfies

$$\eta(K; S_t, n) \begin{cases} \geq 0 & K/S_t \leq e^{n-1}, \\ < 0 & K/S_t > e^{n-1}. \end{cases} \tag{30}$$

Proof. See Appendix A.5. □

To simplify our discussion, we ignore the respective last cases in equations (29) and (30), which refer to the far deep OTM call region $K/S_t > e^{n-1}$. This simplification is innocuous for empirical purposes; call option prices in this deep OTM region are typically negligible, and do not much affect the calculation of the call integral in equation (12).

Equation (29) shows that the η function of the *odd*-degree power-n contract is negative in the put OTM region, and positive in the call OTM region. Therefore, *both* the undervaluation of OTM put options, and the overvaluation of OTM call options, yield an upward bias in the power-3 contract price, which affects the calculation of RNS. Therefore, a positive wedge results in an upward bias in the estimated RNS, via the biases in OTM option prices arising during the extrapolation process. On the other hand, equation (30) shows that the η function of the *even*-degree power-n contract is always non-negative. Consequently, the overvaluation in OTM call prices, and the undervaluation in OTM put prices, yield an offsetting effect on the power-2 and power-4 contract prices, leaving these prices largely unaffected. This explains why MFIV and RNK are largely unaffected by the bias, which arises from extrapolating across BS-IVs.

3.4 A crude RNS measure and robust IV

In the previous section, we showed theoretically that the usage of the BS-IVs contributes to mis-estimating RNS. In this section, we show further that it also biases a popular crude measure of RNS, suggested by Xing et al. (2010) (XZZ). The XZZ measure is defined to be the difference between the standard IVs of an OTM put and that of an ATM call, $XZZ = IV^p_{OTM} - IV^c_{ATM}$; a higher value of XZZ implies that the risk-neutral distribution of the asset returns is skewed to the left. However, the XZZ measure may not be related to RNS, in the case where MR is violated; this is of importance, as we will discuss in Section 5.5. To show this, we decompose XZZ in two components:

$$XZZ = IV^p_{OTM} - IV^c_{ATM} = (IV^p_{OTM} - IV^p_{ATM}) + (IV^p_{ATM} - IV^c_{ATM}) = POMA - IVS, \quad (31)$$

where $POMA = IV^p_{OTM} - IV^p_{ATM}$ is the put out-minus-at IV skew measure (Doran and Krieger (2010) and Fu et al. (2016)), and $IVS = IV^c_{ATM} - IV^p_{ATM}$ is the implied volatility spread (Bali and Hovakimian (2009) and Cremers and Weinbaum (2010), among others).

Under market frictions, the first component may not proxy RNS accurately, whereas the second component introduces noise. In principle, POMA is a measure of the slope of the put IV curve, and thus it could be regarded as a crude measure of RNS. However, Proposition 3.3 shows that, under market frictions, POMA does not satisfy the elementary property that RNS should be zero, even if the underlying price follows a log-normal distribution.[11] Similarly, the IVS component is not related to RNS, given that it is the difference between the call and put option IVs at the *same* strike. Moreover, Proposition 3.3 yields that a non-zero IVS arises in the case where MR is violated; given the property of the cumulative standard normal density function $\Phi(x) + \Phi(-x) = 1$, subtracting equation (26) from (25) yields

$$IV_t^c(K) - IV_t^p(K) \approx \frac{\omega_{t,T}}{R_{t,T}^f} \frac{S_t}{S_t - e^{-r_f \tau} \widetilde{D}_{t,T}} \frac{1}{\sqrt{\tau}\phi(d_1)} = \frac{S_t}{R_{t,T}^f \mathcal{V}_{BS}(K)} \omega_{t,T}, \tag{32}$$

where $\mathcal{V}_{BS}(K) = (S_t - e^{-r_f \tau} \widetilde{D}_{t,T})\phi(d_1)\sqrt{\tau}$ is the BS vega (i.e., the sensitivity of option prices to the change in the volatility parameter). Therefore, a non-zero IVS arises, if and only if the return wedge is non-zero, that is, if and only if MR is violated.[12] This contaminates XZZ further, as proxy of RNS.

However, the XZZ measure will proxy RNS accurately, if it is calculated using the robust IV, rather than the standard IV. First, as we have discussed at the end of Section 3.2, POMA proxies RNS more accurately, once it is calculated based on the robust IV curve. This is because a zero robust IV slope corresponds to the case where the asset price is distributed log-normally. Second, the following Proposition shows that IVS equals zero, once we use the robust IVs, and hence it does not contaminate the XZZ measure as a proxy of RNS.

Proposition 3.4. *Assume that call and put option prices satisfy MR, that is, they satisfy the risk-neutral valuation formulae (equation (B.25)). Let IV_{rob}^c and IV_{rob}^p be the robust IVs calculated from the respective prices of call and put options, which have the same strike price. Then, $IV_{rob}^c = IV_{rob}^p$, regardless of the distribution of the underlying asset.*

Proof. See Appendix A.4. □

[11] Existing literature reports either insignificant predictive power of POMA (Fu et al., 2016), or a positive relation between POMA and future returns (Doran and Krieger, 2010). This suggests that the POMA component should not be the source of a *negative* relation between XZZ and future returns, documented by previous literature.

[12] Note that one can show that equation (32) holds regardless of the distribution of the future underlying price.

4 Data and estimation results

4.1 Dataset

We obtain daily option prices and IVs for January 1996 to December 2017 from the two OptionMetrics Ivy DB (OM) data files, the Volatility Surface (VS) file, and the Standardized Options (SO) file. OM estimate the IV surface of call options and put options separately, and these two files provide the estimated IVs and corresponding European option prices at standardized maturities such as 30, 60, 91, 182, and 365 day-to-maturity. We use 30-day-to-maturity data because subsequently we use the estimated RNS as a sorting variable to construct monthly-rebalanced portfolios. The VS file provides option IVs, option prices, and strike prices at 13 standardized delta-denominated strikes (calls at 0.2, 0.25, ..., 0.8 delta and puts at -0.2, -0.25, ..., -0.8 delta). The SO file provides call and put option prices at the forward at-the-money (ATM), $K = F_{t,T} = e^{r_f \tau} S_t - \widetilde{D}_{t,T}$. We also obtain the underlying price, and the risk-free rate data from the OM database.

We obtain stock return data from CRSP. We also calculate the market beta, firm size, book-to-market, profitability, and investment characteristics from CRSP and Compustat databases. See Appendix C.1, for the detailed description of these characteristics variables. Our universe of individual equities is the U.S. common stocks, which trade at NYSE/Amex/NASDAQ. To obtain this universe, we link the OM database with the CRSP database, and keep only the common stocks (CRSP SHRCD: 10 or 11) traded at NYSE/Amex/NASDAQ (CRSP EXCHCD 1, 2, or 3). Appendix C.2 explains how we link the OM, CRSP, and Compustat databases. Finally, we obtain standard risk factors from the authors' websites.

4.2 Option-implied variables: Estimation

To operationalize the estimation of generalized RNMs, we need a proxy of the return wedge, $\omega_{t,T}$. To this end, we employ the *Spot Synthetic Difference* (SSD) measure proposed by HS. SSD is a properly scaled deviations from put-call parity and HS show that SSD proxies the return wedge, $\omega_{t,T}$, well. We estimate SSD for U.S. individual common stocks traded at NYSE/Amex/NASDAQ, and for the S&P 500 index. To calculate the synthetic stock prices and SSD, we use the pairs of put and call prices with 30 day-to-maturity at the forward ATM strike, recorded in the OM SO file.We back out the dividend payment $\widetilde{D}_{t,T}$ from the forward price recorded in the OM SO file. Appendix C.3 provides the detailed estimation procedure of SSD.[13]

We estimate the RNMs of individual stocks via the original BKM formulae, adjusted for dividend payments (O-RNMs), and via the generalized BKM formulae (G-RNMs), separately. This allows us to examine the effect of the violation of MR in the underlying, to the accurate estimation of RNMs. To estimate the O-RNMs, we set $\omega_{t,T}$ equal to zero in equation (11), and we use the formulae presented in Proposition 3.1; Appendix C.4 provides the details. Note that, strictly speaking, the O-RNMs estimation formulae do not coincide with the original formulae in BKM, due to the modification regarding dividend payments. In Section 5.1, we will document that this modification does not drive our empirical findings.

We estimate G-RNMs, just as we do for O-RNMs, with the difference being that we (i) use the estimated 30-day SSD for $\omega_{t,T}$ in equation (11), and (ii) interpolate the robust IV, instead of the standard IV; our simulation result in Section 3.3 shows that using the standard IV may bias estimated RNMs. To obtain the robust IVs, we use option prices in the SO file, in conjunction with equation (28). In Section 5.1, we disentangle the effects of the IV input, and the employed formulae to the estimation of RNMs. We treat O-RNMs as missing, if the estimated SSD and G-RNMs are missing. We also discard very few month-stock IV observations, if either the calls', or puts' strikes, recorded in the OM VS file, are not strictly monotonic in deltas.

Finally, we calculate two IV slope measures, as follows. First, we calculate the *original* XZZ measure as $XZZ^o = IV^p(-0.2) - IV^c(0.5)$, where $IV^c(d)$ ($IV^p(d)$) stands for the standard call (put) IV, at option's delta equal to d. We obtain the 30-day-to-maturity IVs from the OM VS file. Next, we calculate the *robust* XZZ measure based on the XZZ formula, yet we use the robust IVs, that is, $XZZ^r = IV^p_{rob}(-0.2) - IV^c_{rob}(0.5)$. To obtain the robust IVs, we use equation (28), where we extract the option prices from the corresponding implied volatilities obtained from the OM VS file. Similar to the estimation of RNMs, we treat XZZ^o missing, if the estimated SSD and XZZ^r are missing.

[13]Note that the theoretical formulae to estimate SSD and RNMs rely on European option prices, whereas exchange listed U.S. options are American style. However, the use of the OM VS and SO files circumvent this problem; these files provide the IVs and corresponding European option prices adjusted for the early exercise premium. Our treatment is in line with recent studies using the OM dataset (e.g., Martin and Wagner, 2018).

4.3 Estimated SSD: Do U.S. stocks violate MR?

4.3.1 S&P 500 index

Figure 3a depicts the daily estimated 30-day-to-maturity SSD of the S&P 500 index. We can see that the estimated SSD is highly volatile; the proportion of SSD being positive is about 48% in our sample, that is, the estimated SSD takes positive and negative values with almost the same frequency. This suggests that the $\mathbb{Q}$-expected return of the S&P 500 index is greater, or smaller, than the risk-free rate with almost the same probability. This result is in contrast to the results obtained by previous studies, which test MR via the implied stock approach. These studies find that $S_t^* > S_t$ for more than 90% of samples, which would imply a positive SSD for most of the time; this correspondence between SSD and the implied stock price approach stems from equation (B.20), which holds under the key assumption of the implied stock approach. As we have shown in Section B, the result obtained by the previous literature is likely to reflect a negative IV skew, rather than a violation of MR.

Given the high volatility of the daily SSD, to visualise the pattern of the dynamics of SSD, Figure 3b plots the 21-trading day moving average of the absolute value of the SSD, which measures the degree of the departure of the index from MR (see equation (7)). Two remarks can be drawn. First, SSD takes extreme values periods of market distress, such as the collapse of the IT bubble (around October 2000), and the market meltdown over the recent financial crisis (around October 2008). This is expected given that over these periods, markets frictions, and hence violations of MR, intensify. This pattern is consistent with the findings in HS regarding the SSD of individual stocks; they find that individual stocks tend to take more extreme SSD values during periods where the market is distressed.

Second, apart from the sporadic extreme values of SSD, the S&P 500 index's SSD exhibits a salient downward trend in absolute value terms. In particular, until 2003, the absolute value of SSD lies between 1% to 3% per year, whereas after 2003, it decreased to less than 1% for most of times. This pattern is in line with HS. They show theoretically that the absolute value of SSD is less than the round-trip transaction costs. This implies that the decline in the absolute SSD value is related to a decline in transaction costs. Indeed, transaction costs decreased during early 2000s, due to various institutional reforms.[14]

[Figure 3 about here.]

Next, we assess the statistical significance of the violation of MR. We test the statistical significance of the time-series mean of the *absolute* SSD by means of a 95% ($\pm 2\sigma$) confidence interval; the absolute value of SSD measures the magnitude of the deviation of the risk-neutral expected return from the risk-free rate (equation (7)), that is, it measures the magnitude of the violation of MR.[15]

The 95% confidence interval of the mean value of the absolute SSD is $1.27\% \pm 0.08\%$ per year, when we use the full sample of SSD values. Given the aforementioned salient decline in the absolute SSD value, we also split our sample into two subsamples spanning 1996 to 2002, and 2003 to 2017, respectively. The $\pm 2\sigma$ confidence interval of the former period is $1.98\% \pm 0.1\%$, whereas that of the latter period is $0.94\% \pm 0.06\%$. Therefore, the mean absolute SSD is statistically different from zero in both periods, and it is also economically non-negligible (e.g., compared to the U.S. equity premium, which Fama and French, 2002 estimate as 4.32% per year).

4.3.2 Individual equities

Next, we investigate the SSD of individual equities. We calculate SSD for 12.2 million stock-day observations from 1996 to 2017, for 6,824 distinct stocks. This yields, on average, SSD values for about 2,200 stocks, on each trading day. Table 2, Panel A, reports the summary statistics of the daily SSD. We can see that the SSD exhibits large variations; the standard deviation is 15.9% per year, and the range from fifth to 95th percentiles reaches 31.0% per year. This suggests that the SSD can take an economically large value. Compared to the magnitude of variation, the mean and median of the SSD are

[14]For example, Green et al. (2017) argue that "*the adoption of Reg. FD in October 2000, and the decimalization of quotes in January 2001*" reduced "*effective spreads, price impact, and trading costs.*" They also point out that the introduction of the autoquoting software by NYSE between January and May 2003 "*led to dramatic reductions in trading frictions and costs*" (Green et al., 2017, p.4424). Similarly, French (2008) documents that the per transaction "trading costs" of passive investment, backed out from securities firms' commissions and trading gains data, decreased by half from mid-1990s to 2000s.

[15]We do not examine the mean of the time-series of the raw SSD. This is because a negligible time-series mean SSD should not be interpreted as if the magnitude of SSD is always negligible, and hence that the stock always satisfies MR. For example, the mean of the raw estimated SSD can be close to zero, if it takes large positive and negative SSD values over time, yet with similar frequency. The absolute value of SSD (the scaled deviations from put-call parity) is similar in spirit to the market dislocation index proposed by Pasquariello (2014), in that it is calculated based on the absolute value of deviations from the law of one price.

close to zero (-1.1%, and -0.6% per year, respectively), implying that the individual stock SSD take positive and negative values, with almost the same probability. Regarding the absolute value of SSD, which reflects whether MR holds for individual stocks, the mean and median are 6.8% and 2.8% per year, respectively. This implies that MR is violated for these stocks, that is, market frictions have an economically non-negligible impact on the expected return of individual stocks. Panel B reports the summary statistics of the SSD over the end-of-month trading dates. We can see that the summary statistics are qualitatively the same with those obtained from using the daily SSD.

[Table 2 about here.]

Similar to the S&P 500 index case, we test for each stock, whether MR holds by assessing whether its time-series mean absolute SSD statistically differs from zero. We adopt a critical value of three for the t-test. Thus, we set a high hurdle to reach any conclusions on the significance of SSD. In addition, we record the number of stocks which violate MR, and at the same time their respective absolute SSD is greater than a reference point. This allows us to assess the frequency, as well as the magnitude, of the violation of MR. We consider four alternative values for the reference point: zero, 1%, 2%, and 3% per year (the zero reference point corresponds to testing whether the MR is violated).

The left part of Table 2, Panel C, reports the results where the mean absolute SSD is computed over the daily SSD. We calculate the mean absolute SSD for three groups of stocks, separately, (stocks with at least 21, 1,200, and 2,500 non-missing SSD observations). This serves as a robustness check of any effect of the number of non-missing SSD observation to the calculation of the mean absolute SSD. We can see that the mean absolute SSD is significantly greater than 1% per year for virtually all stocks, regardless of the number of non-missing SSD observations. Moreover, about 75% of four stocks with at least 21 valid observations, have a mean absolute SSD greater than 3% per year. In short, this result suggests that almost all stocks have significantly positive SSD (i.e., violating MR). Moreover, for a large part of stocks, the magnitude of the violation of MR is economically sizeable.

We repeat the same exercise, using only the end-of-month SSD. We report the results in the right part of Panel C. Compared to the daily observations case, there is a smaller proportion of stocks which have statistically significant positive mean absolute

SSD. This is not surprising; the monthly dataset contains far less observations, and hence the standard deviations would be larger. Nevertheless, the implication is the same with the results from the analysis using the daily observations. About 90% of the stocks have a mean absolute SSD greater than 1%, and about 40% of the stocks have a mean absolute SSD greater than 3%.

These results, in conjunction with the findings in HS, strongly suggest that individual stocks' expected returns are affected by market frictions, that is, individual stocks' return wedge is not negligible, and hence individual stocks frequently violate MR.

4.4 Generalized RNMs: Summary statistics

As a preliminary step, for any given RNM, we provide summary statistics, and compute the correlation between O-RNM and G-RNM to assess the effect of the violation of MR in the underlying on the estimated RNMs. We report the summary statistics of end-of-month observations, since we use end-of-month RNMs in the subsequent monthly-rebalancing portfolio analysis. Our untabulated results confirm that there are no qualitative differences in the summary statistics, if we use daily observations.

Table 3, Panel A, reports the descriptive statistics of SSD and the estimated RNMs. We also report the difference between O-RNMs and G-RNMs (e.g., ΔMFIV equals O-MFIV minus G-MFIV). This difference can be interpreted to be the bias in the estimated O-RNMs, resulting from ignoring the violation of MR. We calculate SSD and estimate RNMs for 582,796 month-stock observations, over a 264 months period from January 1996 to December 2017. On average, the SSD and estimated RNMs are available for about 2,200 stocks in each month. The standard deviation of SSD is sizeable, indicating sizeable violations of MR. The median of ΔMFIV, ΔRNS, and ΔRNK are close to zero (-0.02, -0.01, and 0.01, respectively). This implies that the bias in the O-RNMs takes positive and negative sign by almost the same probability. Next, we focus on the ratio of the standard deviation of ΔRNM, and that of the respective G-RNM. This ratio of standard deviations can be interpreted as a rough measure of the "noise-to-signal" ratio; we view the decomposition O-RNM = G-RNM + ΔRNM, as the decomposition of O-RNM into the true RNM and the bias term. The ratio of the MFIV is about 6% ($\sim 1.52/26.68$), that of RNS is about 48% ($\sim 0.23/0.48$), and that of RNK is 34% ($\sim 0.39/1.15$). Therefore, the impact of bias in the estimated O-RNMs is greater for RNS.

Table 3, Panel B, reports the pairwise Pearson and Spearman correlations between O-RNMs and G-RNMs, as well as those between ΔRNM and SSD. We can see that O-RNMs and G-RNMs are almost perfectly correlated for MFIV (1.00 for both Pearson and Spearman coefficients), and for RNK (0.94 and 0.96 for Pearson and Spearman coefficients, respectively). Therefore, the SSD-adjustment has a negligible impact on the estimated MFIV and RNK. On the other hand, in the case of RNS, the two correlation coefficients are 0.90 and 0.89, respectively. Even though these values are still high, in Section 5, we show that the difference between O-RNS and G-RNS creates significant differences in their ability to predict the cross-section of future stock returns.

We can also see that the correlation between SSD and RNMs takes its highest values for O-RNS (0.36 and 0.34), whereas it is almost zero for G-RNS. Moreover, the correlation of SSD with ΔRNM takes its highest value for ΔRNS (0.82, and 0.97). This suggests that the correlation between SSD and O-RNS stems from the correlation between SSD and the "bias" term in the O-RNS, due to the violation of MR, ΔRNS.

[Table 3 about here.]

5 Predictive power of the risk-neutral skewness

In Section 4.4, we have found that the violation of MR in the underlying affects mostly RNS, among the three RNMs. In this Section, we investigate further whether the difference between G-RNS and O-RNS is economically significant, too. We find that O-RNS and G-RNS differ in terms of their respective predictive power for future stock returns. This indicates that the violation of MR is economically significant, for the purposes of computing RNS. We also investigate the predictive power of an alternative measure of RNS, Xing et al.'s (2010) IV slope measure, as a robustness check.

5.1 Predictive power of O- and G-RNS: Baseline analysis

We examine whether O-RNS and G-RNS predict stock returns cross-sectionally, by means of portfolio sort analysis. At the end of each month, we sort stocks into decile portfolios based on the estimated O-RNS and G-RNS, separately. For each estimator of RNS, we form equally-weighted, and value-weighted decile portfolios. We also form the long-short spread portfolios, where we go long in the portfolio with the highest RNS, and go short in the portfolio with the lowest RNS. Table 4 reports the average post-ranking returns of the equally- and value-weighted decile portfolios, formed based on the estimated O-RNS and G-RNS, separately. It also reports the average post-ranking returns, and the risk-adjusted returns (alpha) of the high-minus-low spread portfolios with respect to the CAPM, Fama and French (1993) three-factor model (FF3), Carhart (1997) four-factor model (FFC), the Fama and French (2018) five- and six-factor models (FF5 and FF6), Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY).

Regarding the results based on sorting stocks on O-RNS, we can see an increasing monotonic relation between O-RNS and the decile portfolio returns, for both the equally- and value-weighted portfolios. As a result, the average returns of the equally- and value-weighted long-short portfolios are 114 bps and 79 bps per month, respectively. These values are statistically highly significant (the t-statistics are greater than 3.5). Moreover, the alphas of the spread portfolios are significant and positive; their t-statistics are greater than three in most of the cases, which is the recommended threshold by Harvey et al. (2016); the only exception occurs for α_{FF5} and α_{q4} of the value-weighted spread portfolio. The positive sign of the alphas is in line with the results of recent studies (e.g., Stilger et al., 2017; Borochin and Zhao, 2018; Chordia et al., 2019).

[Table 4 about here.]

On the other hand, the evidence for the predictive power of G-RNS is much weaker, in the case where we sort stocks on G-RNS. The t-statistics of the average returns and alphas do not exceed three (with the exception of α_{SY} of the equally weighted portfolio), which is the threshold value for t-statistics set by Harvey et al. (2016). Moreover, all value-weighted alphas are insignificant at a 1% level, and they are insignificant even at a 5% level, except α_{FFC}. These results suggest that the G-RNS does not predict future stock returns, given the recent literature on addressing data snooping concerns, which recommends a high threshold t-statistics value (e.g., Harvey et al., 2016), and a value-weighted portfolio construction (e.g., Bali et al., 2016; Hou et al., 2018).

Table 5 reports the factor loadings obtained from regressing the returns of the value-weighted spread portfolios sorted by O-RNS and G-RNS, separately, on a number of specifications of factor models. We can see that there is no qualitative change in the magnitude of the factor loadings between the O-RNS-sorted and G-RNS-sorted spread portfolios. Thus, the significant change in alphas of the long-short portfolio, when switching from the O-RNS to the G-RNS sorting criterion, is not caused by the change in the exposure of the two portfolios to standard risk-factors.

[Table 5 about here.]

The difference in the predictive ability of O-RNS and G-RNS may stem from two sources: the different formulae used to compute RNS, and/or the difference in the IVs used at the interpolation and extrapolation stage. We disentangle the effect from these two sources by estimating RNS in four different ways: OS-, OR-, GS-, and GR-RNS. The first letter of the prefix stands for the type of the formula ("Original" or "Generalized" BKM formula). The second letter of the prefix stands for the type of the IVs ("Standard" IVs or "Robust" IVs).Then, we construct long-short spread portfolios by sorting stocks into decile portfolios, based on these four alternative RNS estimates. Figure 4 shows the t-statistics of the average return and alphas of the long-short spread portfolios. We can draw three remarks. First, the magnitude of t-statistics decreases from OS-, OR-, GS-, to GR-RNS. This pattern suggests that both channels (the RNS formula and the IV) contribute to the decrease in the predictive power of RNS, in line with our simulation result in Section 3.3. Second, in the case of the theoretically consistent with market frictions GS-RNS, the average return and all alphas, but FFC, become statistically insignificant, when we apply the criteria to address data mining concerns. Third, this insignificance becomes more pronounced when GR-RNS is used. These findings suggest that the previously documented ability of O-RNS (OS-RNS) to predict stock returns, arises because the violation of MR had not been taken into account in both the RNS formula, as well as in the computation of IVs.

[Figure 4 about here.]

5.2 Predictive power of O- and G-RNS: Robustness tests

We conduct further robustness tests to assess whether the predictive power of RNS ceases to exist, once we take into account the violation of MR. First, we run Fama and MacBeth (1973) (FM) regressions. At the end of each month, we conduct a cross-sectional regression, where we regress the stock returns during the succeeding month on the estimated

RNS, and other firms' and stocks' characteristics. Then, we report the time-series average of the estimated cross-sectional intercepts, as well as the coefficients of the characteristic variables with their respective t-statistics. At any point in time, we conduct two types of regressions for the cross-sectional regression: the ordinary least square (OLS) regression, and the value-weighted least square (VWLS) regressions employed by Green et al. (2017). OLS- and VWLS-based FM regressions can be viewed as the counterparts of the equally-weighted, and value-weighted portfolios in the portfolio analysis, respectively. Therefore, given the recent literature on data-snooping concerns, it is desirable to examine the statistical significance of O-RNS and G-RNS, under the VWLS regression, too (see also a related discussion in Hou et al., 2018).

Table 6 reports the results from the FM regressions. Columns (1) to (4) report the results, including O-RNS as a regressor. The results from the two univariate regressions of O-RNS, Columns (1) and (2), show that O-RNS is a statistically significant predictor of stock returns. Moreover, Columns (3) and (4) show that this predictive power remains, even after controlling for standard characteristic variables. These results corroborate our portfolio sort result based on O-RNS, as well as the finding in the previous literature on the predictive ability of O-RNS (e.g., Stilger et al., 2017; Borochin and Zhao, 2018).

Next, we examine the predictive power of G-RNS in the FM setting. Columns (5) and (6) show that G-RNS does not predict stock returns in the univariate regression, in line with our finding from the portfolio sort analysis. The results in the multivariate regressions are mixed; G-RNS becomes significant under the OLS regression (Column (7)), whereas it remains insignificant under the VWLS regression (Column (8)). These results are analogous to our portfolio sort result; we find that several alphas of equally-weighted G-RNS spread portfolio are significant, whereas those of value-weighted G-RNS spread portfolio are not. Therefore, the results from the FM regressions suggest that G-RNS does not predict future stock returns, once we apply criteria which address data snooping concerns.

[Table 6 about here.]

Second, we repeat the portfolio sort analysis of Section 5.1 by forming quintile, instead of decile portfolios. Each quintile portfolio contains twice as many stocks as a decile portfolio. As a result, quintile portfolios are more diversified, and hence less affected by possible outliers. Table 7 reports the results. We can see that the O-RNS-sorted spread portfolios earn positive and significant average return and alphas, whereas the G-RNS-sorted spread portfolios do not earn significant returns, just as it was in the case when decile portfolios were formed. This suggests that our baseline result is not driven by possible outliers.

[Table 7 about here.]

Finally, we examine whether our treatment of dividend payments in the estimation of RNS affects its predictability. So far, we have found that O-RNS predicts stock returns, using a modified BKM RNS formula which takes dividends into account. This is in line with the finding of the previous literature, which uses the original BKM formulae, whose derivation assumes non-dividend paying stocks. To further confirm that the (lack of the) predictability of O-RNS (G-RNS) is robust to the treatment of dividend payments, we estimate RNS by ignoring the dividend payment adjustment; we re-estimate the O- and G-RNS by setting $\widetilde{D}_{t,T} = 0$ (i.e., assuming all stocks pay no dividends,), and repeat the portfolio sort analysis. Even though this is a theoretically imprecise treatment, this exercise would expedite the comparison between our results and those in the previous literature, which does not consider dividend payments in the calculation of RNS.

Table 8 reports the results. As we can see, there are no qualitative changes in the predictive power of the O-RNS and G-RNS; O-RNS predicts future returns, whereas the G-RNS does not. This result suggests that our findings on the (lack of the) predictability of O-RNS (G-RNS), as well as the findings in the previous literature are not affected by whether dividend payments are taken into account to estimate RNS.

[Table 8 about here.]

We also estimate the RNS of the *ex-dividend return*, using Proposition 3.2. Then, we repeat the portfolio sorting analysis described in Section 5.1, by using the estimated O- and G-RNS of the ex-dividend returns. Our untabulated results do not differ from the results obtained from the baseline analysis. This was expected because, under the deterministic dividend assumption, the distribution of the ex-dividend return coincides with that of the cum-dividend return shifted by a deterministic amount, and hence the higher order moments do not change.[16]

[16]Note that the moments of the ex-dividend and cum-dividend *log* returns may differ due to the non-

5.3 SSD and the predictive power of O-RNS

So far, we have confirmed that O-RNS predicts future returns, whereas G-RNS does not. We conjecture that O-RNS predicts future returns solely because its estimation error, ΔRNS, predicts stock returns. This is because ΔRNS arises from the violation of MR, that is, from the presence of market frictions manifested by the violation of MR. Therefore, ΔRNS is expected to be correlated with SSD. HS find that SSD predicts stock returns, thus ΔRNS should also predict stock returns. This conjecture is in line with the lack of the predictability of G-RNS; G-RNS is free from the bias caused by the violation of MR, and this explains the lack of correlation between G-RNS and SSD, as the results in Table 3 have shown.

We confirm our conjecture by means of three tests. First, we repeat the portfolio sorting analysis, where we sort stocks in decile portfolios using ΔRNS as a sorting variable. Given the almost perfect rank correlation between ΔRNS and SSD (Table 3), we effectively sort stocks based on SSD, which HS document that it has predicts stock returns.[17] Table 9 shows the results. In line with the predictive ability of SSD, we can see that ΔRNS predicts stock returns. Moreover, its predictive power is stronger than that of O-RNS; t-statistics of the equally-weighted spread returns and alphas reach a value of nine, and those of the value-weighted ones range between values of four to five. This result implies that the predictive power of O-RNS is "condensed" in the ΔRNS part, given the reported inability of G-RNS to predict stock returns.

[Table 9 about here.]

Next, we investigate the relation between the predictive power of O-RNS and the degree of the violation of MR. If the predictive power of O-RNS stems from the violation of MR, we expect that O-RNS will not have predictive power among stocks which do not severely violate MR (i.e., SSD is close to zero). On the other hand, we expect that the predictive power of O-RNS will be stronger among stocks which severely violate MR. We examine these conjectures by means of a dependent bivariate portfolio sort. We first sort stocks into quartile portfolios based on the absolute value of SSD; the absolute value of

linearity of the logarithm function. However, this effect is negligible, given a typical dividend-to-stock price ratio.

[17] HS document that the alphas of the SSD-sorted value-weighted decile spread portfolio are about 180 bps per month, and their t-statistics are greater than five. This result suggests that the return predictive power of SSD is stronger than that of O-RNS reported in Table 4.

SSD measures the degree of the violation of MR. Then, within each one of four subgroups, we form quartile portfolios by sorting stocks based on O-RNS, and we calculate the post-ranking average return of the spread portfolios (highest O-RNS minus lowest O-RNS).

Table 10 reports the average returns and alphas of the highest O-RNS minus lowest O-RNS spread portfolios, for any given portfolio constructed on absolute SSD; we consider equally weighted and value weighted portfolios. The results confirm our two conjectures. The average returns and alphas are greatest for the highest |SSD| bin, and lowest for the lowest |SSD| bin. Therefore, O-RNS exhibits stronger predictive power among the stocks which are subject to severer violations of MR. Specifically, the results for the lowest |SSD| bin (the first and fifth Columns) indicate that O-RNS does not predict future stock returns among the stocks in this bin; all returns and alphas, but α_{SY} of the equally-weighted portfolio, are insignificant at 5% level (α_{SY} of the equally-weighted portfolio is insignificant at a 1% level).[18] In line with our conjecture, this result suggests that O-RNS does not predict stock returns, when MR is not (severely) violated. Its predictive power increases with the level of absolute SSD, thus confirming that the predictive power of O-RNS reflects the presence of market frictions; in the case where there were no frictions, O-RNS would not predict future stock returns.

[Table 10 about here.]

Finally, we provide direct evidence that the predictive power of O-RNS stems from SSD by employing the *SSD-adjusted return regression* proposed by HS. HS show that in the case where the pricing kernel m is described by a linear combination of risk factors $\boldsymbol{f}$, the intercept α of the following regression of the SSD-adjusted excess return on risk factors,

$$R_{t,T} - R^f_{t,T} - SSD_{t,T} = \alpha + \boldsymbol{\beta}' \boldsymbol{f}_T + \varepsilon_T, \tag{33}$$

should be zero. We examine this hypothesis for each one of the O-RNS-sorted value-weighted decile portfolios, and for the associated long-short spread portfolio. Failing to reject this hypothesis, would suggest that the significant alphas of the SSD-*non*-adjusted returns of the O-RNS-sorted portfolios, reported in Table 4 (Panel B), stem from the presence of market frictions, that is, from a non-zero SSD.

[18] We also conduct three-by-three and five-by-five double sorting exercises, and we confirm that results do not change qualitatively.

Table 11 reports the results, where we regress the SSD-adjusted returns of each one of the O-RNS sorted value-weighted decile portfolios, as well as that of the long-short spread portfolio, on a set of factors. In line with the positive correlation between SSD and O-RNS, the first row shows that the portfolio average SSD is monotonically increasing. However, the alphas (intercepts) of the SSD-adjusted excess returns of decile portfolios are almost always insignificant. In addition, the spread portfolio's alphas are now insignificant, even though the non-SSD adjusted case (Table 4, Panel B) exhibits significant alphas. The fact that the SSD-adjustment removes the significant predictive power of O-RNS implies that the predictive power of the O-RNS is driven by the SSD component, i.e. by the effect of market frictions on the expected returns, and it is not driven by (omitted) risk factors. This corroborates our previous finding that the predictability of the O-RNS stems from its correlation with SSD.

[Table 11 about here.]

5.4 Predictability of O-RNS: Implications of our findings

In this subsection, we explore further implications of our findings with respect to the ongoing debate on the mechanism to explain the predictive power of O-RNS (limits-of-arbitrage versus informed option trading explanation). Our findings, so far, support the limits-of-arbitrage mechanism as an explanation of the predictive power of O-RNS, and they corroborate previous evidence that the predictability of O-RNS is stronger among stocks which face relatively larger market frictions (e.g., Stilger et al. (2017), Chordia et al. (2019)).

However, our approach to explain the predictive power of O-RNS is distinct from that taken by the existing limits-of-arbitrage-based explanations. First, we acknowledge that market frictions violate MR, and we provide a new formula to estimate RNS. The previous literature is subject to an inconsistency: it explains the predictive power of O-RNS by recognising that frictions exist, yet it uses the O-RNS BKM formula, which is derived under the assumption that no frictions exist. Second, our SSD-based explanation can accommodate both the negative and positive informational content of O-RNS, with respect to future stock returns.[19] This is in contrast to the explanations based on short-

[19] Our empirical findings indicate that the SSD, and the associated bias in O-RNS, take both positive and negative values with roughly the same probability. Given that SSD has an "alpha" interpretation

sale constraints and downside risk (e.g., Stilger et al. (2017), and Gkionis et al. (2018)), which can explain negative informational content of O-RNS only.

We examine further whether the predictive power of O-RNS is more pertaining to the market frictions, than the informed option trading explanation. Under the informed option trading story, we expect that the prices of options (and hence RNS) with zero option trading volume, should have less informational content (i.e., lower predictive power), than these of traded options. To examine this conjecture, on each end-of-month trading day, we group stocks into two subgroups: stocks whose options have non-zero aggregate trading volume on that date, and remaining zero option trading stocks.[20] Within any given option trading volume category, we sort stocks in decile value-weighted portfolios, based on O-RNS. We calculate the monthly post ranking returns of these portfolios, as well as these of the spread portfolio.

Table 12 reports the results. We can see that the average return and alphas of the spread portfolio are greater for the non-traded options subgroup. The informed option trading and option trading demand stories are at odds with this empirical pattern. On the other hand, the friction-based story can explain the greater predictive power among non-option trading stocks. Table 12 shows that stocks with non option trading are smaller, have wider bid-ask spread, and are less liquid compared to those with non-zero option trading, that is, stocks with zero option trading tend to face larger market frictions. In line with these stock characteristics, the portfolio average SSD of the short (long) leg takes more negative (positive) value among the non-traded subgroup, implying that these stocks are affected more by the violation of MR than those with non-zero option trading. Our finding is analogous with Goncalves-Pinto et al. (2017), who find that their option-based return predictor, termed DOTS, does not exhibit a different level of predictability among stocks with zero and non-zero option trading volume.

[Table 12 about here.]

(i.e., the expected stock return adjusted for the covariance risk premium, see HS), this implies that O-RNS can signal both future outperformance and underperformance, depending on the sign of SSD.

[20] We use the aggregate option trading volume, across all strikes and maturities of both call and put options, as the trading activity indicator, for the following reasons. First, the IVs, and corresponding option prices, are obtained from the interpolated volatility surfaces. As a result, there are no trading volume data for each option price and IV. Second, OM estimates volatility surfaces using all option data over all available strikes and maturities. The aggregate trading volume is a consistent measure with this practice. Finally, we focus on a short-term horizon. Hence, the aggregate trading volume is a good proxy of the horizon of our interest, since short-term options are relatively heavily traded.

Two final remarks are in order regarding the debate on the mechanism of the predictive ability of O-RNS. First, admittedly, the validity of the limits-of-arbitrage story for the O-RNS predictive power mechanism is not accepted universally. For example, Chordia et al. (2019) conduct dependent bivariate portfolio sort exercises, where they first control for either Amihud's (2002) illiquidity measure, or the idiosyncratic volatility (IVOL), and then they sort stocks on O-RNS in bins. They find that O-RNS retains its predictive power, even in the lowest Amihud and IVOL bins, that is, among stocks which face low degree of limits-of-arbitrage. Thus, they conclude that limits-of-arbitrage cannot fully account for the RNS-return relation. However, our result in Table 10 suggests the opposite story. Our bivariate portfolio sort analysis resembles that of Chordia et al. (2019), except that we use the absolute value of SSD to measure the degree of the limits-of-arbitrage. In our analysis, we find that O-RNS does not predict stock returns in the bin with the smallest frictions. Our results are based on the use of a theoretically-founded measure of the aggregate effect of the limits-of-arbitrage, whose correlation with specific types of frictions is imperfect (HS).

Second, our results in Table 12, albeit supportive for the limits-of-arbitrage story, do not preclude informed option trading, as an explanation of other empirical predictive patterns. Nevertheless, the existence of market frictions is still a prerequisite to invoke the informed option trading explanation. The theoretical model of Easley et al. (1998) predicts that informed traders trade in the option market, only when there are sufficiently relatively bigger frictions in the stock market. In our case, the non-zero SSD, which gives rise to the predictive power of O-RNS, indicates that market frictions exist, even in the presence of informed option traders. This is because we calculate SSD based on the scaled deviations from put-call parity. In the absence of market frictions, SSD would become zero because arbitrageurs would exploit deviations from put-call parity instantaneously.

5.5 Predictive power of implied volatility slope

In Section 3.4, we showed that the XZZ BS-IV slope measure, XZZ^o, may not accurately proxy RNS, in the case where MR is violated. In this case, we showed that the robust IV-based XZZ measure, XZZ^r, should be used, instead. In this subsection, we further document that the predictive power of the XZZ measure vanishes, once we calculate it based on the robust IVs. This is not surprising; the XZZ measure calculated with the robust IV curve is the counterpart of G-RNS. Both the robust IV and G-RNS take the possible violation of MR in the underlying asset into account.

Table 13, Panel A, reports the summary statistics of SSD, XZZ^o, XZZ^r, estimated at the end of each month, and their difference $\Delta XZZ = XZZ^o - XZZ^r$. We can see that SSD and the XZZ measures are calculated for about 2,200 stocks on average. The mean and median of the two XZZ measures are both positive, implying that the IV curve exhibits a negative skew more often than a positive skew, regardless of using the BS IV, or the robust IVs. Table 13, Panel B, reports the pairwise Pearson and Spearman rank correlations. The correlation between the original and robust XZZ measures is fairly high (0.8). SSD and the original XZZ measure are negatively correlated, whereas SSD and the robust XZZ measure do not show strong correlation. These patterns are in line with our discussion in Section 3.4; the original XZZ measure is mechanically negatively correlated with IVS and SSD (equations (31) and (32)), whereas the robust XZZ measure is not affected by IVS because the robust IVS is zero.

[Table 13 about here.]

Next, we sort stocks in ascending order, based on XZZ^o and XZZ^r, separately, and we form value-weighted decile portfolios. Then, we construct the long-short spread portfolios, where we go long in the stocks with the highest XZZ measures, and short in the stocks with the lowest measures. Table 14 reports the result. First, consistent with Xing et al. (2010), the original XZZ measure negatively predicts future stock returns; the decile portfolio returns are generally decreasing in the level of the XZZ measure. The average return and alphas of the long-short spread portfolio are highly significant; t-statistics are above 5.35 (3.71) in magnitude for the equally-weighted portfolio (value-weighted portfolio). On the other hand, XZZ^r does not predict stock returns. The average returns and alphas are all insignificant at a 5%-significance level for the value-weighted portfolio, and mostly insignificant for the equally-weighted portfolio.

[Table 14 about here.]

We repeat the SSD-adjusted regression considered in Section 5.3 (Table 11) for the XZZ-sorted portfolios, to examine whether the predictive power of XZZ^o stems from that of SSD. The dependent variables in these regressions are the SSD-adjusted returns of the XZZ^o-sorted value-weighted decile portfolios, and that of their long-short spread portfolio. Table 15 reports the result. We can see that the average portfolio SSD of decile portfolios are monotonically decreasing in the level of the XZZ^o, in line with its mechanical relation to SSD, equation (31). The alpha (intercept) of the SSD-adjusted excess returns are overall insignificant. This is in contrast to the non-SSD adjusted case (Table 14, Panel B), where alphas were significant. Similar to the O-RNS case, the switch from the significant alphas to insignificant alphas, implies that the predictive power of the XZZ^o is driven by the SSD component.

[Table 15 about here.]

In sum, even though the standard and robust XZZ measures are highly correlated, there is an economically significant difference in the their predictive power. The original XZZ measure of RNS loses its ability to predict stock returns, once we account for the possible violation of MR in the estimation of IVs. This is analogous to our findings that G-RNS does not predict stock returns, and showcases the need to also use the robust IVs, in the case where MR is violated.

6 Conclusions

Risk-neutral moments (RNMs) is the building block in the vast literature which explores the informational content of market option prices. This literature estimates RNMs by assuming that market frictions in the underlying asset do not exist. We revisit the estimation of RNMs, and we provide novel formulae to estimate them, in the presence of market frictions. Our RNM formulae generalize the seminal Bakshi et al. (2003) (BKM) formulae.

We relate the estimation of RNMs to market frictions by relying on the concept of the martingale restriction (MR). The violation of MR, that is, the deviation of the expected risk-neutral return from the risk-free rate, implies the presence of market frictions. Our formulae differ from BKM, in that the risk-neutral expected return is modified. Under the violation of MR, the risk-neutral expected asset returns deviate from the risk-free rate, and our generalized formulae take into account such a return wedge. Moreover, to increase the accuracy in the estimated RNMs, we propose the concept of robust implied volatility (IV), which preserves desirable properties (e.g., returning flat IV curves when the distribution follows log-normal) even when MR is violated.

By employing the Spot Synthetic Difference (SSD) measure of Hiraki and Skiadopoulos (2023) (HS) as a proxy of the return wedge, we find that both the S&P 500, and the U.S. individual equities, frequently violate MR. This finding motivates us to investigate the effect of market frictions to the estimated RNMs. Our empirical analysis shows that the difference between the risk-neutral skewnesss (RNS) calculated by the original BKM formula (O-RNS), and that calculated by our generalized BKM formula (G-RNS) is economically significant. O-RNS predicts the cross-section of future stock returns in line with the previous literature, whereas G-RNS does not; this discrepancy becomes more apparent when robust IVs are used to estimate either G-RNS, or a proxy of RNS. We document that the predictive power of O-RNS stems from its estimation bias, caused by the violation of MR, and it is stronger when calculated from stocks which underlie options with lower rather than bigger option trading volume. Interestingly, our approach to estimate RNMs under frictions, allows us to contribute to the ongoing debate on the mechanism of the predictive power of O-RNS. Our results strongly suggest that the predictive power of O-RNS stems from the existence of limits-to-arbitrage due to market frictions.

Our study has revealed that the estimation bias in RNMs can be economically significant in the context of RNS. However, our theoretical results are relevant to any studies on the information content of option prices, which rely on the validity of MR, including the estimation of risk-neutral distributions, empirical pricing kernels, risk-aversion parameters, and portfolio allocation, among others. These topics are best left for future research.

A Proofs

A.1 Proof of Proposition 3.1

Carr and Madan (2001) show that any twice differentiable payoff function $f(S_T)$ satisfies

$$f(S_T) = f(F) + f'(F)(S_T - F) + \int_F^\infty f''(K)(S_T - K)^+ dK + \int_0^F f''(K)(K - S_T)^+ dK, \tag{A.1}$$

for an arbitrary positive number F. The power-n log return contract functions satisfy $f(S_t - \widetilde{D}_{t,T}) = f'(S_t - \widetilde{D}_{t,T}) = 0$. Hence,

$$f(S_T) = \int_{S_t - \widetilde{D}_{t,T}}^{\infty} f''(K)(S_T - K)^+ dK + \int_0^{S_t - \widetilde{D}_{t,T}} f''(K)(K - S_T)^+ dK. \tag{A.2}$$

We multiply both sides by $e^{-r_f \tau}$, and take the risk-neutral expectation. Under the assumption that option prices satisfy MR, we obtain:

$$e^{-r_f \tau}\mathbb{E}_t^{\mathbb{Q}}[f(S_T)] = \int_{S_t - \widetilde{D}_{t,T}}^{\infty} f''(K)C_t(K,T)dK + \int_0^{S_t - \widetilde{D}_{t,T}} f''(K)P_t(K,T)dK. \tag{A.3}$$

Taking the second derivatives for $f(S_T) = \log(S_T + \widetilde{D}_{t,T}/S_t)^n$, we obtain equations (12) and (13).

Next, note that $e^{r_f\tau} + \omega_{t,T} = \mathbb{E}_t^{\mathbb{Q}}[R_{t,T}] = \mathbb{E}_t^{\mathbb{Q}}[e^{r_{t,T}}]$ holds. Therefore, expanding the last expression as a forth-order Taylor series approximation, yields

$$e^{r_f\tau} + \omega_{t,T} = 1 + \mathbb{E}_t^{\mathbb{Q}}[r_{t,T}] + \frac{1}{2}\mathbb{E}_t^{\mathbb{Q}}[(r_{t,T})^2] + \frac{1}{6}\mathbb{E}_t^{\mathbb{Q}}[(r_{t,T})^3] + \frac{1}{24}\mathbb{E}_t^{\mathbb{Q}}[(r_{t,T})^4] + o((r_{t,T})^4). \tag{A.4}$$

In line with BKM, we rearrange, and ignore the higher-order residual term to obtain

$$\widetilde{\mu}_{t,T} := \mathbb{E}_t^{\mathbb{Q}}[r_{t,T}] = e^{r_f\tau} + \omega_{t,T} - 1 - \frac{e^{r_f\tau}}{2}M(2)_{t,T} - \frac{e^{r_f\tau}}{6}M(3)_{t,T} - \frac{e^{r_f\tau}}{24}M(4)_{t,T}. \tag{A.5}$$

This proves equation (11). Then, equations (8) to (10) follow. □

A.2 Proof of Proposition 3.2

Since the payoff functions change to $f(S_T) = \log(S_T/S_t)^n$ for the ex-dividend case, we now have $f(S_t) = f'(S_t) = 0$. Therefore, the boundary value for the two integrals in equations (A.2) and (A.3) changes from $S_t - \widetilde{D}_{t,T}$ to S_t. Moreover, we obtain (14) and (15), by calculating the second derivative of the ex-dividend power-n log return functions. Next, note that the following relation holds:

$$\mathbb{E}_t^{\mathbb{Q}}[\exp(r_{t,T}^{ex})] = \mathbb{E}_t^{\mathbb{Q}}\left[\frac{S_T}{S_t}\right] = \mathbb{E}_t^{\mathbb{Q}}[R_{t,T}] - \frac{\widetilde{D}_{t,T}}{S_t} = e^{r_f\tau} + \omega_{t,T} - \frac{\widetilde{D}_{t,T}}{S_t}. \tag{A.6}$$

Expanding the left hand side of equation (A.6) by a Taylor series approximation similar to equation (A.4), and rearranging, yields equation (16). □

A.3 Proof of Proposition 3.3

First, we prove (a) for the call option case. Let $c(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, \sigma, r_S)$ be the call pricing function, equation (20), under the distribution (17). Our purpose is to find the expression of $IV^c(K; r_S)$, which solves $c(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, \sigma, r_S) = BS_{call}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, IV^c(K; r_S))$. In what follows, we suppress all arguments of the c function and IV^c, but r_S.

The first-order Taylor series approximation of the (standard) BS call function with respect to the volatility parameter around the true volatility parameter σ, yields

$$BS_{call}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, IV^c(r_S)) \approx BS_{call}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, \sigma) + \mathcal{V}_{BS}(IV^c(r_S) - \sigma), \quad \text{(A.7)}$$

where $\mathcal{V}_{BS}$ is the BS vega evaluated at $(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, \sigma)$. On the other hand, the first-order Taylor series approximation of the c function with respect to the r_S-argument around $r_S = r_f$ yields

$$c(r_S) \approx c(r_f) + \left.\frac{\partial c}{\partial r_S}\right|_{r_S = r_f} (r_S - r_f). \quad \text{(A.8)}$$

The left-hand side of equations (A.7) and (A.8) are the same by the definition of the IV, $c(r_S) = C_t(K, T) = BS_{call}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, IV^c(r_S))$. Moreover, the first terms in the right-hand side of equations (A.7) and (A.8) are also the same; $c(r_f) = BS_{call}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, \sigma)$ follows from equation (20). Therefore, equations (A.7) and (A.8) yield

$$IV^c(r_s) \approx \sigma + \frac{r_S - r_f}{\mathcal{V}_{BS}} \left.\frac{\partial c}{\partial r_S}\right|_{r_S = r_f}. \quad \text{(A.9)}$$

Calculation of the partial derivative $\partial c / \partial r_s$ yields

$$\begin{aligned} \frac{\partial c}{\partial r_S} &= e^{(r_S - r_f)\tau} \left(\tau BS_{call}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, \sigma) + \left.\frac{\partial BS_{call}(S_t, K, \tau, r, \widetilde{D}_{t,T}, \sigma)}{\partial r}\right|_{r = r_S} \right) \\ &= e^{(r_S - r_f)\tau} \tau S_t \Phi(d_1), \end{aligned} \quad \text{(A.10)}$$

because the *rho* of the BS function with discrete dividends evaluated at $r = r_S$ is given by $\tau(S_t\Phi(d_1) - BS_{call}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, \sigma))$. Therefore, equation (A.9) reduces to

$$IV^c(r_S) \approx \sigma + (r_S - r_f)\tau \frac{S_t \Phi(d_1)}{\mathcal{V}_{BS}} = \sigma + (r_S - r_f)\tau \frac{S_t}{S_t - e^{-r_f\tau}\widetilde{D}_{t,T}} \frac{\Phi(d_1)}{\sqrt{\tau}\phi(d_1)}, \quad \text{(A.11)}$$

because the BS vega is given by $\mathcal{V}_{BS} = \left(S_t - e^{-r_f\tau}\widetilde{D}_{t,T}\right)\phi(d_1)\sqrt{\tau}$. Since $\omega_{t,T}/R^f_{t,T} = \mathbb{E}^{\mathbb{Q}}_t[R_{t,T}]/R^f_{t,T} - 1 = e^{(r_S - r_f)\tau} - 1 \approx (r_S - r_f)\tau$ holds, equation (25) follows. A similar calculation for the put option, proves equation (26).

Next, we prove (b). The statement about the inequality relation between σ and IV^c or IV^p is obvious because $\Phi(x)/\phi(x)$ is always positive. To prove the remaining claim, it suffices to show $\Phi(x)/\phi(x)$ is increasing in x. Since the derivative $(\Phi(x)/\phi(x))'$ is given by

$$\frac{\phi^2(x) - \Phi(x)\phi'(x)}{\phi^2(x)} = \frac{\phi^2(x) + x\Phi(x)\phi(x)}{\phi^2(x)} = \frac{\phi(x) + x\Phi(x)}{\phi(x)}, \tag{A.12}$$

it suffices to show $\phi(x) + x\Phi(x) > 0$, for any x. This is obvious for $x \geq 0$. For $x < 0$, we prove that $x > -\phi(x)/\Phi(x)$. In this case, $\phi(x)/\Phi(x)$ (the inverse Mills ratio of the standard normal distribution) is known to be equal to the negative of the truncated mean $-\mathbb{E}[Z|Z < x]$, where Z is a standard normal random variable (see e.g., Theorem 22.2 of Greene, 2003). Then, $x > \mathbb{E}[Z|Z < x]$ follows. □

A.4 Proof of Proposition 3.4

The observed put and call prices satisfy the following relation.

$$\begin{aligned} & e^{-(r_S - r_f)\tau} C_t(K,T) - e^{-(r_S - r_f)\tau} P_t(K,T) = e^{-(r_S - r_f)\tau} e^{-r_f\tau} \mathbb{E}_t^{\mathbb{Q}}[S_T - K] \\ &= e^{-r_S\tau} \mathbb{E}_t^{\mathbb{Q}}[S_T - K] = e^{-r_S\tau} S_t \mathbb{E}_t^{\mathbb{Q}}[R_{t,T}] - e^{-r_S\tau} \widetilde{D}_{t,T} - e^{-r_S\tau} K \\ &= (S_t - e^{-r_S\tau} \widetilde{D}_{t,T}) - e^{-r_S\tau} K. \end{aligned} \tag{A.13}$$

Therefore, it follows that $IV_{rob}^c = IV_{rob}^p = IV_{rob}$ if and only if

$$BS_{call}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, IV_{rob}) - BS_{put}(S_t, K, \tau, r_S, \widetilde{D}_{t,T}, IV_{rob}) = (S_t - e^{-r_S\tau}\widetilde{D}_{t,T}) - e^{-r_S\tau} K. \tag{A.14}$$

Equation (A.14) holds because of the property of the BS formulae with deterministic dividends. □

A.5 Proof of Lemma 3.1

For simplicity, we assume that the stock pays no dividends. The weighting function $\eta(K; S_t, n)$ in equation (12) equals

$$\eta(K; S_t, n) = \frac{n(\log(K/S_t))^{n-2}}{K^2} \left[(n-1) - \log(K/S_t)\right]. \tag{A.15}$$

For strikes $K/S_t > e^{n-1}$, equation (A.15) is negative regardless of whether n is odd, or even. This proves the respective last inequalities in equations (29), and (30).

For strikes $K/S_t < e^{n-1}$, the square bracket term in equation (A.15) is positive. Therefore, the sign of the η function equals the sign of $(\log(K/S_t))^{n-2}$. This term is always non-negative, when n is an even number, proving equation (30). On the other hand, when n is an odd number, this term is non-negative for $K/S_t \geq 1$, and negative for $K/S_t < 1$. This proves equation (29). □

B On the implied stock price approach

B.1 Theoretical considerations

Longstaff (1995) studies first whether a stock price satisfies MR empirically by testing whether equation (2) holds. To this end, he assumes that the unobservable right-hand side of equation (2) can be estimated by the *implied stock price* S_t^* extracted from the observed market option prices, that is, he assumes that

$$S_t^* = \frac{1}{R_{t,T}^f}\mathbb{E}_t^{\mathbb{Q}}[S_T + \widetilde{D}_{t,T}]. \tag{B.16}$$

Under this assumption, Longstaff (1995), and subsequent studies, typically test MR by investigating whether the relative difference between the implied stock price and the observed stock price, Δ_t, is different from zero:

$$\Delta_t = \frac{S_t^* - S_t}{S_t}. \tag{B.17}$$

To extract the implied stock price, first, an option pricing model $h(\xi, S_t, K, \tau, r_f, \widetilde{D}_{t,T})$, where ξ denotes the set of parameter(s), is selected. A typical choice in the literature is the Black and Scholes (1973) (BS) model, where ξ consists of one parameter, the volatility. Then, S_t^* is calculated, jointly with the model parameters in ξ, by minimizing the sum of squared errors between the observed option prices $O_t(K,T)$, and model-based option prices $h(\xi, S_t^*, K, \tau, r_f, \widetilde{D}_{t,T})$:

$$\min_{\xi, S_t^*} \sum \left[O_t(K,T) - h(\xi, S_t^*, K, \tau, r_f, \widetilde{D}_{t,T})\right]^2. \tag{B.18}$$

We explain the relation between the implied stock price and the wedge term $\omega_{t,T}$ in equation (7). Adding and subtracting $\mathbb{E}_t^{\mathbb{Q}}[S_T + \widetilde{D}_{t,T}]/R_{t,T}^f$ to the numerator of equation (B.17), and using equation (7) yield

$$\Delta_t = \frac{S_t^* - \mathbb{E}_t^{\mathbb{Q}^*}[S_T + \widetilde{D}_{t,T}]/R_{t,T}^f}{S_t} + \frac{\omega_{t,T}}{R_{t,T}^f}. \tag{B.19}$$

Therefore, if the assumption made by Longstaff (1995), equation (B.16) holds, then Δ_t and the wedge $\omega_{t,T}$ are proportionally related as follows

$$\Delta_t = \frac{\omega_{t,T}}{R_{t,T}^f}. \tag{B.20}$$

Therefore, for the Longstaff (1995) approach to be a valid way to test MR, the validity of his assumption, equation (B.16), is crucial. Unfortunately, this assumption does not hold in general. To see this point, we consider the most frequently considered situation in the literature, where the option pricing model is set to the BS model. The following Proposition shows the conditions under which the implied stock price approach, based on the BS option pricing model, constitutes a valid test of MR.

Proposition B.1. *Assume that options satisfy MR, yet the underlying stock may violate MR. Assume further that (i) the true $\mathbb{Q}$-distribution of S_T is log-normal, and (ii) there are at least two observed option prices as an input to the minimization problem described in equation (B.18). Then,*

$$\Delta_t^{BS} := \frac{S_{t,BS}^* - S_t}{S_t} = \frac{\omega_{t,T}}{R_{t,T}^f}, \tag{B.21}$$

where $S_{t,BS}^*$ is given as the solution of equation (B.18), in the case where the option pricing model used to extract the implied stock price is the BS model.

Proof. Let m and σ be the mean and the volatility parameters of the true log-normal distribution, respectively, that is, $S_T \sim \text{Lognormal}(m, \sigma^2\tau)$. The BS benchmark model assumption suggests that the benchmark model-based distribution of S_T is given by

$$\text{Lognormal}\left(\log(S_t^* - e^{-r_f\tau}\widetilde{D}_{t,T}) + r_f\tau - \frac{(\sigma^*)^2\tau}{2}, (\sigma^*)^2\tau\right). \tag{B.22}$$

When there are two or more observed option prices in the minimization problem, equation (B.18), the two parameters of the true distribution, m and σ, can be correctly identified and satisfy $\sigma = \sigma^*$ and

$$m = \log(S_t^* - e^{-r_f\tau}\widetilde{D}_{t,T}) + r_f\tau - \frac{(\sigma^*)^2\tau}{2}. \tag{B.23}$$

In this case, the expected stock price under the true risk-neutral probability measure is calculated as

$$\mathbb{E}_t^{\mathbb{Q}}[S_T] = \exp\left(m + \frac{\sigma^2\tau}{2}\right) = e^{r_f\tau}S_t^* - \widetilde{D}_{t,T}, \tag{B.24}$$

due to equation (B.23). Equation (B.24) yields $S_t^* = e^{-r_f\tau}\mathbb{E}_t^{\mathbb{Q}}[S_T + \widetilde{D}_{t,T}]$. Hence, equation (B.16) holds. This proves equation (B.21). □

Proposition B.1 shows that the implied stock price approach constitutes a valid test of MR (i.e., MR holds, if and only if $\Delta_t^{BS} = 0$), *if* the true $\mathbb{Q}$-distribution follows a log-normal distribution, and the BS model is used to extract the implied stock price. On the other hand, we show by means of simulation later in this appendix section that the BS model-based implied stock price approach spuriously rejects MR even if MR holds, if the true $\mathbb{Q}$-distribution of the future stock price is not log-normal.

B.2 Implied stock approach: Simulation

We explore the ability of the implied stock approach to test MR, under a simulation setting, where MR holds. The simulation setup is as follows. We choose $S_t = 100$, $\tau = 1/6$ and $r_f = 4\%$. For dividends, we set $\widetilde{D}_{t,T} = 0.5$, which corresponds to about 3% annualized dividend yield. Call and put options trade at strikes $K = 80, 85, \ldots, 120$ (i.e., nine strikes). We assume that both the underlying stock and options satisfy MR under the true $\mathbb{Q}$ measure and thus option prices are given by the risk-neutral valuation formulae

$$C_t(K,T) = e^{-r_f\tau}\mathbb{E}_t^{\mathbb{Q}}[(S_T - K)^+], \quad \textit{and} \quad P_t(K,T) = e^{-r_f\tau}\mathbb{E}_t^{\mathbb{Q}}[(K - S_T)^+], \tag{B.25}$$

where $(x)^+ = \max(x, 0)$. We assume that the $\mathbb{Q}$-distribution of the stock price may not be log-normal. Hence, the BS implied volatility (BS-IV) curve may be *skewed*, that is, the graph of the BS-IV as a function of the option moneyness may not be flat. We model the BS-IV curve as a linear function of moneyness $IV(K/S_t) = \sigma_{ATM} + k(K/S_t - 1)$. We set the at-the-money (ATM) volatility, σ_{ATM} to 20%. The coefficient k controls the slope of the IV skew. We examine five realistic values for the slope coefficient k: $k = \pm 1/2$, $k = \pm 1/4$, and $k = 0$. For example, $k = -1/2$ corresponds to the situation where the BS-IV=30% at $K = 80$ (i.e., moneyness = 0.8), and BS-IV=10% at $K = 120$ (i.e., moneyness = 1.2). Then, we convert the BS-IV at each traded strike to the respective call and put option prices, using the BS option pricing functions with deterministic dividend payments, that is,

$$C_t(K,T) = BS_{call}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, IV) = (S_t - e^{-r_f\tau}\widetilde{D}_{t,T})\Phi(d_1) - e^{-r_f\tau}K\Phi(d_2), \tag{B.26}$$

where Φ is the cumulative density function of the standard normal distribution, and

$$d_1 = \frac{\log\left(\frac{S_t - e^{-r_f\tau}\widetilde{D}_{t,T}}{K}\right) + (r_f + \frac{1}{2}IV^2)\tau}{IV\sqrt{\tau}}, \quad and \quad d_2 = d_1 - IV\sqrt{\tau}. \tag{B.27}$$

The put option price is given by

$$P_t(K,T) = BS_{put}(S_t, K, \tau, r_f, \widetilde{D}_{t,T}, IV) = e^{-r_f\tau}K\Phi(-d_2) - (S_t - e^{-r_f\tau}\widetilde{D}_{t,T})\Phi(-d_1). \tag{B.28}$$

This construction ensures that the underlying satisfies MR, too. To show this, $C_t(K) - P_t(K) = S_t - e^{-r_f\tau}\widetilde{D}_{t,T} - e^{-r_f\tau}K$ follows from the definition of BS-IV via the BS functions (equations (B.26), and (B.28)). $C_t(K) - P_t(K) = e^{-r_f\tau}\mathbb{E}_t^{\mathbb{Q}^*}[S_T - K]$ follows from equation (B.25). These two equations yield $S_t = e^{-r_f\tau}\mathbb{E}_t^{\mathbb{Q}}[S_T + \widetilde{D}_{t,T}]$.[21] Note that to ensure that the underlying satisfies MR, it is crucial to convert the *same* BS-IV to obtain the call and put option prices; in Section 3.4, we show that the violation of MR is equivalent to the non-zero implied volatility spread (i.e., the difference between the call BS-IV and put BS-IV extracted from call and put options with the same strike). Given these generated option prices, we obtain the two parameters, the implied stock price S_t^* and the BS volatility parameter σ^*, by solving equation (B.18).

Table 16 reports the simulation result. In line with the literature on testing MR, we estimate S_t^* and σ^* using a set of call options and a set of put options, separately. We also report the estimate of the (annualized) return wedge as $\widehat{\omega_{t,T}} = R_{t,T}^f\widehat{\Delta}_t/\tau$, under the implied stock price approach (equation (B.20)). The implied stock price approach is an alternative way to estimate the underlying stock return wedge, only once its key assumption, equation (B.16), is imposed. We can see that the two parameters, S_t^* and σ^*, are correctly estimated under $k = 0$. In this case, the implied stock price approach identifies correctly that MR holds. This is in line with Proposition B.1 because $k = 0$ corresponds to a situation, where the true $\mathbb{Q}$-distribution follows a log-normal distribution. On the

[21] Note that we make no assumption that the BS model is the true model; we simply use it as a translation mechanism to map IVs to option prices. In fact, the designed simulation admits that the BS model is not the true model, unless $k = 0$.

contrary, when k is negative (positive), S_t^* is estimated to be lower (higher) than S_t, regardless of whether one uses call or put options. Therefore, even though we assume that the underlying satisfies MR, the implied stock price approach spuriously rejects MR, when the slope of the IV curve is non-zero.

[Table 16 about here.]

We explain the mechanism which yields $S_t^* > S_t$, in the case where $k < 0$ (i.e., negative IV skew case). The implied stock price S_t^* is chosen so that the BS-IVs of the *model-based* option prices $h(\xi, S_t^*, K, \tau, r_f, \widetilde{D}_{t,T})$ fit the negatively skewed BS-IVs of the *observed* option prices, as close as possible. For a set of call options, for this to occur, in-the-money (ITM) calls should be more "expensive" (in terms of the BS-IV) compared to out-of-the-money (OTM) calls. To achieve this, the implied stock price approach sets S_t^* higher than S_t. This is equivalent to setting a positive $\widehat{\omega_{t,T}}$ (equation (B.20)), that is, setting the estimate of the expected stock return under $\mathbb{Q}$ greater than the risk-free rate (equation (7)). Therefore, choosing $S_t^* > S_t$ makes call options more expensive by setting the expected stock return higher. In addition, this effect is stronger for ITM call options than OTM call options. This is because one can benefit from a higher expected stock price only in the case where call options expire in-the-money; this is more likely for ITM options than OTM options. Figure 5a shows the effect of choosing a higher value for S_t^*. The BS model-based IV with a higher S_t^* locates closer to the true simulated IV curve, compared to the "initial" BS model-based IV curve, where the parameters are set to $S_t^* = S_t$ and $\sigma^* = \sigma_{ATM}$ (the gray thin line). Finally, σ^* is estimated lower than the average IV level (i.e., σ_{ATM}), to offset increases in call option prices due to a higher S_t^*, which would prevent fitting the level of the given IV curve. Figure 5b shows the effect of choosing a lower value for σ^*; the BS model-based IV curve with lower σ^* locates closer to the true simulated IV line, compared to the BS model-based IV curve with $\sigma^* = \sigma_{ATM}$.

For the case of put options, a negative IV skew means that OTM puts are more "expensive" (in terms of the IV) compared to OTM puts. To create such a pattern, again a higher S_t^* is chosen by the minimisation algorithm. This is because choosing $S_t^* > S_t$ makes put options cheaper. This effect is stronger for ITM put options than OTM ones (Figure 5c) for an analogous reason to the call case. Finally, σ^* is estimated higher than σ_{ATM} to offset decreases in put option prices due to a higher S_t^* which again would prevent fitting the level of the IV curve (Figure 5d).

[Figure 5 about here.]

Three remarks are in order regarding the results of our simulation. First, the results are in line with the empirical results reported in the literature; Longstaff (1995), and the follow-up studies, report that the estimated stock price is greater than the current stock price (i.e., $S_t^* > S_t$), in more than 90% of the cases in their samples. Even though the previous literature views this as evidence for the violation of MR, our simulation results suggest that the reported violation of MR, under the implied stock approach, may be a reflection of the negative IV skew, typically encountered in index options.

Second, our simulated results show that the implied stock approach rejects MR spuriously, in the case where the BS model is chosen to calculate the implied stock price, and an IV skew exists. In other words, even if $\omega_{t,T}$ equals zero, Δ_t^{BS} may not equal zero. This implies that the key assumption of the implied stock price approach, equation (B.16), may not supported by the data. In addition, the spurious rejection of MR under the implied stock price approach may also be a result of the selected (possibly mis-specified) option pricing model. This manifests that the implied stock price approach is subject to testing a joint hypothesis: it tests a property of the true risk-neutral distribution (i.e., MR) by assuming a specific model for the risk-neutral distribution.

Finally, that the Longstaff (1995) approach does not constitute a valid MR test does not mean MR holds in general. On the contrary, testing MR under the SSD-approach circumvents the joint hypothesis problem, as well as the strong assumption shown by equation (B.16).

C Description of dataset and variables

In this Appendix, we provide a detailed explanation for the data sources and the calculation procedures for the characteristic variables, SSD, and RNMs. We also explain how we link the three databases we use: OM, CRSP and Compustat.

C.1 Firms' and stocks' characteristic variables

Beta: In each month, we regress daily stock excess returns over past 12 months on the daily excess market return, to estimate the beta. We require there are at least 200 non-missing observations. We obtain stock return data from the CRSP database. We use the excess market return, provided at Kenneth French's website.

SIZE: Size is the natural logarithm of the market equity. The market equity is calculated as the product of the number of outstanding share with the price of the stock at the end of each month. We obtain data from the CRSP database.

Book-to-Market equity (B/M): We follow Davis et al. (2000) to measure book equity as stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (Compustat annual item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if it is available. If not, we measure stockholders' equity as the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock. From June of each year t to May of $t+1$, we caculate the book-to-market equity (B/M) as the ratio of the book equity for the fiscal year ending in calendar year $t-1$ to the market equity at the end of December of year $t-1$. We treat non-positive B/M data as missing.

Profitability: We follow Fama and French (2015) to measure profitability as revenues (Compustat annual item REVT) minus cost of goods sold (item COGS) if available, minus selling, general, and administrative expenses (item XSGA) if available, minus interest expense (item XINT) if available, divided by (non-lagged) book equity. From June of year t to May of $t+1$, we assign profitability for the fiscal year ending in calendar year $t-1$.

Investment: We follow Fama and French (2015) to measure investment as the change in total assets (Compustat annual item AT) from the fiscal year ending in year $t-1$ to the fiscal year ending in t, divided by $t-1$ total assets. From June of year t to May of $t+1$, we assign investment for the fiscal year ending in calendar year $t-1$.

C.2 Linking OM, CRSP, and Compustat databases

In our analysis, we use three databases, OM, CRSP, and Compustat. To link CRSP and Compustat, we use the SAS linking macro provided by the Wharton Research Data Services (WRDS), and match PERMNO (the CRSP identifier) and GVKEY (the Compustat identifier). To link CRSP and OM, we link SECID (the OM identifier) and PERMNO by the SAS linking macro provided by the WRDS. This WRDS macro links SECID and PERMNO based on the CUSIP code, the similarity in the issuer's (company) name recorded in OM and CRSP, and partly the security ticker. The macro also provides the score of the strength of the link based on a 0–7 integer scale (0 stands for the strongest link, where the eight-digit CUSIP exactly matches as well as the issuer's name matches). We keep SECID-PERMNO link with the score less than or equal to three. After linking SECID, GVKEY and PERMNO, we keep only U.S. common stocks (SHRCD 10 or 11) traded in NYSE/Amex/NASDAQ (EXCHCD 1, 2, or 3).

C.3 Estimation of SSD

HS show that the underlying stocks' SSD satisfies the following approximate relation:

$$SSD_{t,T} \approx \frac{R^0_{t,T}}{S_t}(\widetilde{S}_{t,T}(K) - S_t), \tag{C.29}$$

where $\widetilde{S}_{t,T}(K)$ is the *synthetic stock* price defined as

$$\widetilde{S}_{t,T}(K) = C_t(K,T) - P_t(K,T) + \frac{K + \widetilde{D}_{t,T}}{R^f_{t,T}}. \tag{C.30}$$

Their result, equation (C.29), shows that SSD is accurately proxied by the observable *scaled* deviations from put-call parity.

To estimate SSD, we use an alternative equivalent expression of $\widetilde{S}_{t,T} - S_t$ in the right-hand side of equation (C.29).

$$\widetilde{S}_{t,T}(K) - S_t = BS_{call}(S_t, K, \tau, r, \widetilde{D}_{t,T}, IV^c_t(K)) - BS_{call}(S_t, K, \tau, r, \widetilde{D}_{t,T}, IV^p_t(K)), \tag{C.31}$$

where BS_{call} is the Black-Scholes European call option function with deterministic dividend payment (equation (B.26)), and $IV^c_t(K)$ ($IV^p_t(K)$) is the IV of the call (put) option. We estimate the right-hand side of equation (C.29) by calculating deviations from put-call parity based on equation (C.31).

To this end, we use the 30-day forward ATM call and put IVs provided by the OM SO file, that is, $K = F_{t,T} = e^{r_f \tau} S_t - \widetilde{D}_{t,T}$ and $\tau = 30$ days. We obtain the underlying stock price S_t and the risk-free rate r_f from the OM database. We interpolate the OM zero yield data to obtain the risk-free rate, for the specified day-to-maturity. Regarding the dividend payment $\widetilde{D}_{t,T}$, we back it out from the forward price in the OM SO file as $\widetilde{D}_{t,T} = e^{r_f \tau} S_t - F_{t,T}$.

C.4 Estimation of risk-neutral moments

We estimate risk-neutral moments (RNMs) based on the formulae shown in Section 3. We obtain the option price data across a wide range of strike from the OM VS file. The OM VS file contains the smoothed IVs for standardized maturities and deltas. It contains IVs and associated option prices and strike prices, at delta equals 0.2, 0.25, ..., 0.8, for call options, and -0.8, -0.75, ..., -0.2 for put options, for 30, 60, 91, 122, 152, 182, 273, 365, 547 and 730 calendar days-to-maturity. We use the 30-day-to-maturity data to estimate RNMs. Therefore, this file allows us to use call and put option data for 13 different strikes, for standardized time-to-maturities. The OM volatility surface file provides IVs, implied strike prices, and implied European option prices at each standardized delta for call and put options, separately. We discard day-stock observations, and hence do not estimate the RNMs, when the strike prices provided by the OM of calls or puts are not monotonic in deltas. Such a situation may occur when the interpolated volatility surface exhibits an extremely steep skew.

We follow Stilger et al. (2017), to interpolate and extrapolate IVs as follows. We interpolate call IVs and put IVs, separately, in the strike-IV dimension, using a cubic piecewise Hermite polynomial interpolation scheme. Then, we extrapolate horizontally the IV beyond the highest and the lowest strikes. To estimate O-RNMs, we interpolate the IVs provided by OM. To estimate G-RNMs, first we calculate the robust IVs from the implied option prices provided by OM. Then, we interpolate these robust IVs. Once we obtain the continuous IV curve, we calculate option prices at 1001 equally-spaced strike points over the moneyness from 1/3 to 3. Finally, we estimate O-RNMs and G-RNMs by numerically calculating the the integrals showing in the original BKM formulae, and Proposition 3.1, respectively.

The usage of the OM VS file facilitates our empirical analysis in two ways. First, it provides the IVs at a standardized maturity, and for a unified strike range (0.2 to 0.8 deltas in the absolute value) across stocks. This helps us to avoid any issues arising from employing RNMs estimated from different maturities, and different moneyness ranges covered by traded options. Second, this dataset has been used in the estimation of RNMs (e.g., Borochin and Zhao, 2018; Chordia et al., 2019), thus making possible the comparison of our results to theirs.

References

Amihud, Y. (2002): "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects," *Journal of Financial Markets*, 5, 31–56.

Ammann, M. and A. Feser (2019): "Robust Estimation of Risk-Neutral Moments," *Journal of Futures Markets*, forthcoming.

Asquith, P., P. A. Pathak, and J. R. Ritter (2005): "Short Interest, Institutional Ownership, and Stock Returns," *Journal of Financial Economics*, 78, 243–276.

Bakshi, G. S., N. Kapadia, and D. Madan (2003): "Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity," *Review of Financial Studies*, 16, 101–143.

Bali, T. G., R. F. Engle, and S. Murray (2016): *Empirical Asset Pricing: The Cross Section of Stock Returns*, Wiley.

Bali, T. G. and A. Hovakimian (2009): "Volatility Spreads and Expected Stock Returns," *Management Science*, 55, 1797–1812.

Bali, T. G., J. Hu, and S. Murray (2018): "Option Implied Volatility, Skewness, and Kurtosis and the Cross-Section of Expected Stock Returns," .

Black, F. and M. Scholes (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637–654.

Bliss, R. R. and N. Panigirtzoglou (2002): "Testing the stability of implied probability density functions," *Journal of Banking and Finance*, 26, 381–422.

Bollen, N. P. and R. E. Whaley (2004): "Does Net Buying Pressure Affect the Shape of Implied Volatility Functions?" *Journal of Finance*, 59, 711–753.

Borochin, P., H. Chang, and Y. Wu (2018): "The Information Content of the Term Structure of Risk-Neutral Skewness," .

Borochin, P. and Y. Zhao (2018): "What Information Does Risk Neutral Skewness Contain? Evidence From Momentum Crashes," .

Brunnermeier, M. K. and L. H. Pedersen (2009): "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, 22, 2201–2238.

Carhart, M. M. (1997): "On Persistence in Mutual Fund Performance," *Journal of Finance*, 52, 57–82.

Carr, P. and D. Madan (2001): "Optimal positioning in derivative securities," *Quantitative Finance*, 1, 19–37.

Çetin, U., R. Jarrow, P. Protter, and M. Warachka (2006): "Pricing options in an extended black scholes economy with illiquidity: Theory and empirical evidence," *Review of Financial Studies*, 19, 493–529.

Chabakauri, G. (2013): "Dynamic equilibrium with two stocks, heterogeneous investors, and portfolio constraints," *Review of Financial Studies*, 26, 3104–3141.

Chen, J., H. Hong, and J. C. Stein (2002): "Breadth of Ownership and Stock Returns," *Journal of Financial Economics*, 66, 171–205.

Chordia, T., T.-C. C. Lin, and V. Xiang (2019): "Risk-neutral Skewness and Informed Trading," .

Christoffersen, P., K. Jacobs, and B. Y. Chang (2013): "Forecasting with Option-Implied Information," in *Handbook of Economic Forecasting, Volume 2*, ed. by G. Elliott and A. Timmermann, Elsevier, 581–656.

Cochrane, J. H. (2005): *Asset Pricing, Revised Edition*, Princeton University Press.

Conrad, J., R. F. Dittmar, and E. Ghysels (2013): "Ex Ante Skewness and Expected Stock Returns," *Journal of Finance*, 68, 85–124.

Cremers, M. and D. Weinbaum (2010): "Deviations from Put-Call Parity and Stock Return Predictability," *Journal of Financial and Quantitative Analysis*, 45, 335–367.

Davis, J. L., E. F. Fama, and K. R. French (2000): "Characteristics, Covariances, and Average Returns: 1929 to 1997," *Journal of Finance*, 55, 389–406.

Dennis, P. and S. Mayhew (2009): "Microstructural biases in empirical tests of option pricing models," *Review of Derivatives Research*, 12, 169–191.

Doran, J. S. and K. Krieger (2010): "Implications for asset returns in the implied volatility skew," *Financial Analysts Journal*, 66, 65–76.

Drechsler, I. and Q. F. Drechsler (2014): "The Shorting Premium and Asset Pricing Anomalies," .

Easley, D., M. O'Hara, and P. S. Srinivas (1998): "Option Volume and Stock Prices: Evidence on Where Informed Traders Trade," *Journal of Finance*, 53, 431–465.

Faccini, R., E. Konstantinidi, G. Skiadopoulos, and S. Sarantopoulou-Chiourea (2019): "A New Predictor of U.S. Real Economic Activity: The S&P 500 Option Implied Risk Aversion," *Management Science*, 65, 4927–4949.

Fama, E. F. and K. R. French (1993): "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 33, 3–56.

——— (2002): "The Equity Premium," *Journal of Finance*, 47, 427–465.

——— (2015): "A Five-Factor Asset Pricing Model," *Journal of Financial Economics*, 116, 1–22.

——— (2018): "Choosing Factors," *Journal of Financial Economics*, 128, 234–252.

Fama, E. F. and J. D. MacBeth (1973): "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy*, 81, 607–636.

Frazzini, A. and L. H. Pedersen (2014): "Betting against beta," *Journal of Financial Economics*, 111, 1–25.

French, K. R. (2008): "Presidential address: The cost of active investing," *Journal of Finance*, 63, 1537–1573.

Fu, X., Y. E. Arisoy, M. B. Shackleton, and M. Umutlu (2016): "Option-Implied Volatility Measures and Stock Return Predictability," *Journal of Derivatives*, 24, 58–78.

Gârleanu, N. and L. H. Pedersen (2011): "Margin-based Asset Pricing and Deviations from the Law of One Price," *Review of Financial Studies*, 24, 1980–2022.

Gârleanu, N., L. H. Pedersen, and A. M. Poteshman (2009): "Demand-Based Option Pricing," *Review of Financial Studies*, 22, 4259–4299.

Giamouridis, D. and G. Skiadopoulos (2011): "The Informational Content of Financial Options for Quantitative Asset Management: A Review," in *The Oxford Handbook of Quantitative Asset Management*, ed. by B. Scherer and K. Winston, Oxford University Press, 243–265.

Gkionis, K., A. Kostakis, G. Skiadopoulos, and P. S. Stilger (2018): "Positive Stock Information in Out-of-the-Money Option Prices," .

Goncalves-Pinto, L., B. D. Grundy, A. Hameed, T. van der Heijden, and Y. Zhu (2017): "Why Do Option Prices Predict Stock Returns? The Role of Price Pressure in the Stock Market," .

Green, J., J. R. M. Hand, and X. F. Zhang (2017): "The Characteristics that Provide Independent Information about Average U.S. Monthly Stock Returns," *The Review of Financial Studies*, 30, 4389–4436.

Greene, W. H. (2003): *Econometric Analysis*, Prentice Hall, fifth edit ed.

Guo, B., Q. Han, and D. Ryu (2013): "Is the KOSPI 200 Options Market Efficient? Parametric and Nonparametric Tests of the Martingale Restriction," *Journal of Futures Markets*, 33, 629–652.

Harrison, J. M. and D. M. Kreps (1979): "Martingales and arbitrage in multiperiod securities markets," *Journal of Economic Theory*, 20, 381–408.

Harrison, J. M. and S. R. Pliska (1981): "Martingales and Stochastic Integrals in the Theory of Continuous Trading," *Stochastic Processes and their Applications*, 11, 215–260.

Harvey, C. R., Y. Liu, and H. Zhu (2016): "... and the Cross-Section of Expected Returns," *Review of Financial Studies*, 29, 5–68.

He, H. and D. M. Modest (1995): "Market Frictions and Consumption-Based Asset Pricing," *Journal of Political Economy*, 103, 94–117.

Hentchel, L. (2003): "Errors in Implied Volatility Estimation," *Journal of Financial and Quantitative Analysis*, 38, 779–810.

Hiraki, K. and G. Skiadopoulos (2023): "The Contribution of Transaction Costs to Expected Stock Returns: A Novel Measure," *Journal of Derivatives*, 31, 8–33.

Hou, K., C. Xue, and L. Zhang (2015): "Digesting Anomalies: An Investment Approach," *Review of Financial Studies*, 28, 650–705.

——— (2018): "Replicating Anomalies," *Review of Financial Studies*, forthcoming.

Huang, H. H., K. Wang, and Z. Wang (2016): "A test of efficiency for the S&P 500 index option market using the generalized spectrum method," *Journal of Banking and Finance*, 64, 52–70.

Leland, H. E. (1985): "Option Pricing and Replication with Transactions Costs," *Journal of Finance*, 40, 1469–1484.

Longstaff, F. A. (1995): "Option Pricing and the Martingale Restriction," *Review of Financial Studies*, 8, 1091–1124.

Luttmer, E. G. J. (1996): "Asset Pricing in Economies with Frictions," *Econometrica*, 64, 1439–1467.

Martin, I. and C. Wagner (2018): "What is the Expected Return on a Stock?" .

Neumann, M. and C. Schlag (1996): "Martingale Restrictions and Implied Distributions for German Stock Index Option Prices," .

Ofek, E., M. Richardson, and R. F. Whitelaw (2004): "Limited Arbitrage and Short Sales Restrictions: Evidence from the Options Markets," *Journal of Financial Economics*, 74, 305–342.

Pasquariello, P. (2014): "Financial market dislocations," *Review of Financial Studies*, 27, 1868–1914.

Rehman, Z. and G. Vilkov (2012): "Risk-Neutral Skewness: Return Predictability and Its Sources," .

Stambaugh, R. F. and Y. Yuan (2017): "Mispricing Factors," *Review of Financial Studies*, 30, 1270–1315.

Stilger, P. S., A. Kostakis, and S.-H. Poon (2017): "What Does Risk-Neutral Skewness Tell Us About Future Stock Returns?" *Management Science*, 63, 1814–1834.

Strong, N. and X. Xu (1999): "Do S&P 500 Index Options Violate the Martingale Restriction?" *Journal of Futures Markets*, 19, 499–521.

Xing, Y., X. Zhang, and R. Zhao (2010): "What Does the Individual Option Volatility Smirk Tell Us About Future Equity Returns?" *Journal of Financial and Quantitative Analysis*, 45, 641–662.

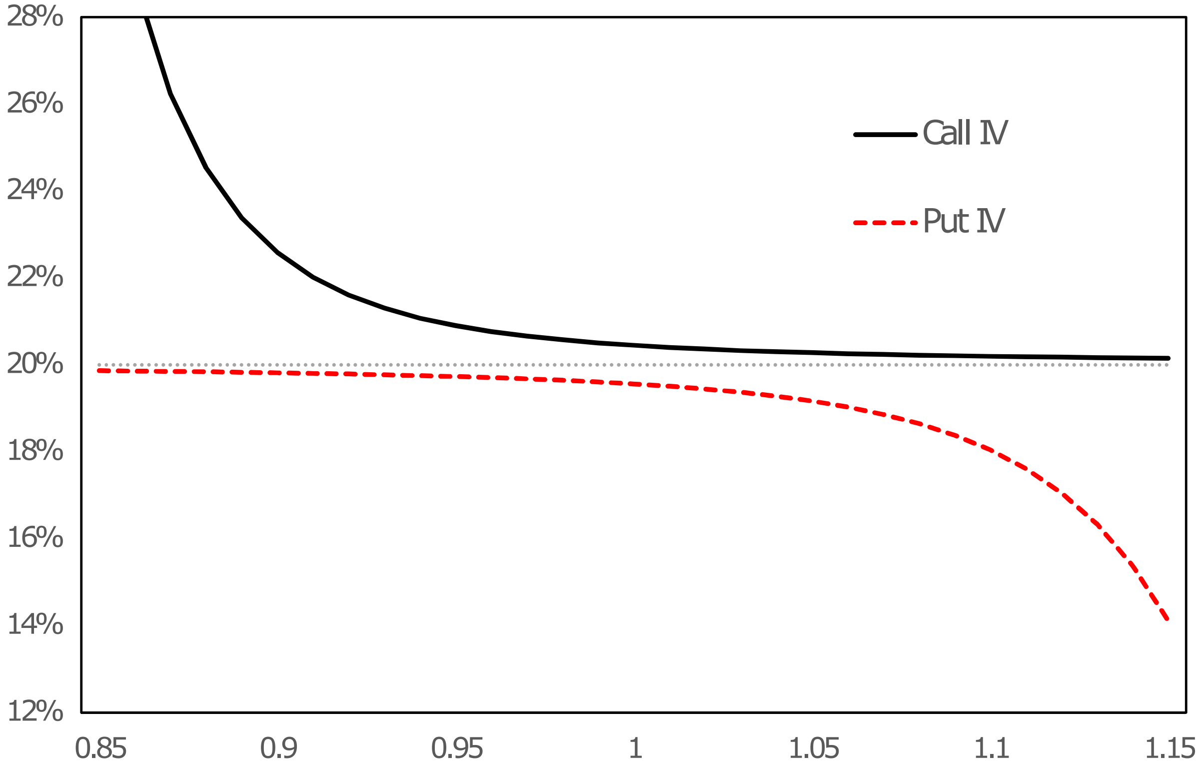


Figure 1. Implied volatility and the violation of MR

This Figure depicts the call and put BS-IV curves as a function of option's moneyness (strike to stock price ratio) based on equations (25) and (26), respectively. The time-to-maturity is $\tau = 1/8$, the risk-free rate is $r_f = 3\%$, the volatility parameter is $\sigma = 20\%$, the deterministic dividend-to-stock price is $\widetilde{D}_{t,T}/S_t = 0.5\%$ and the annualized SSD is $SSD_{t,T}/\tau = 1\%$. The x-axis is the moneyness, and the y-axis is the implied volatility in percentage.

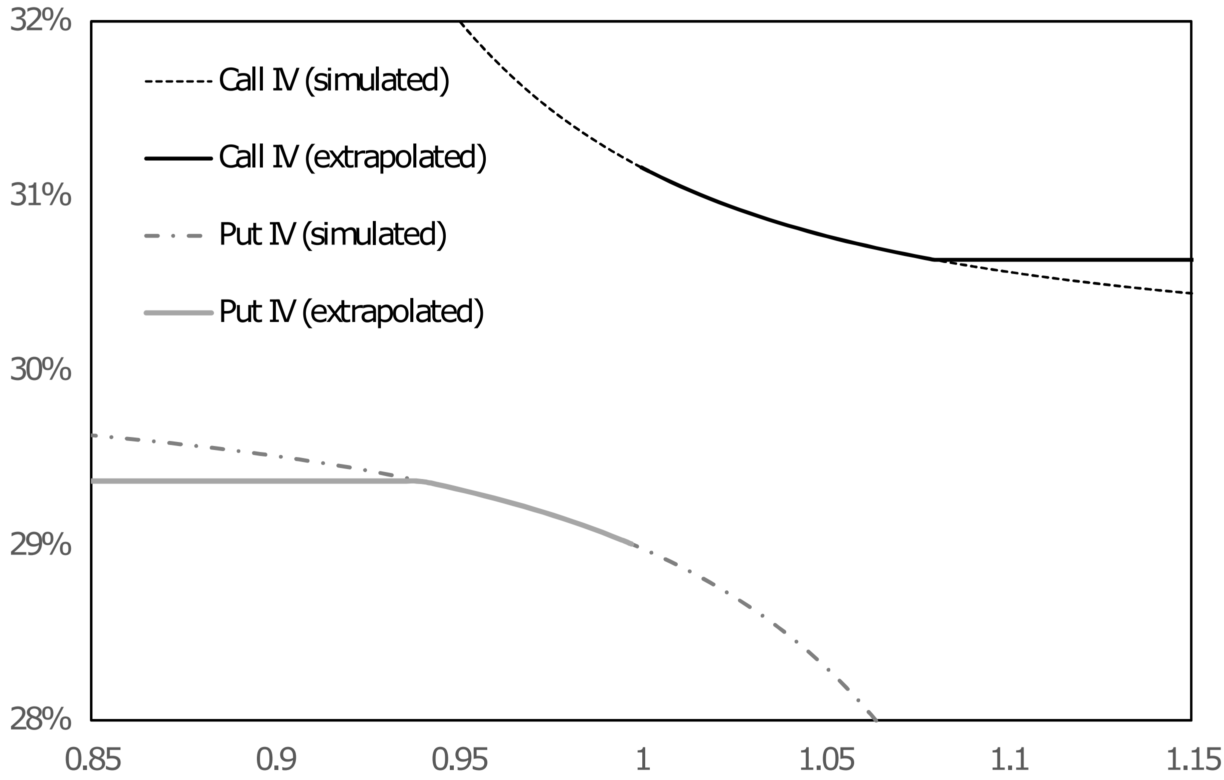
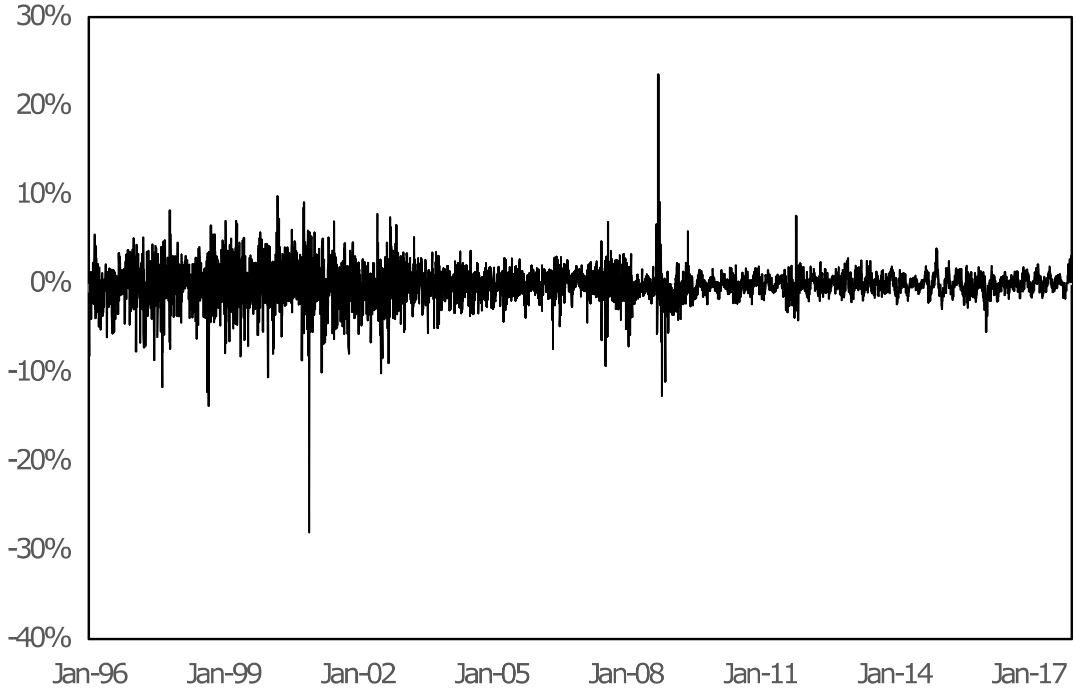
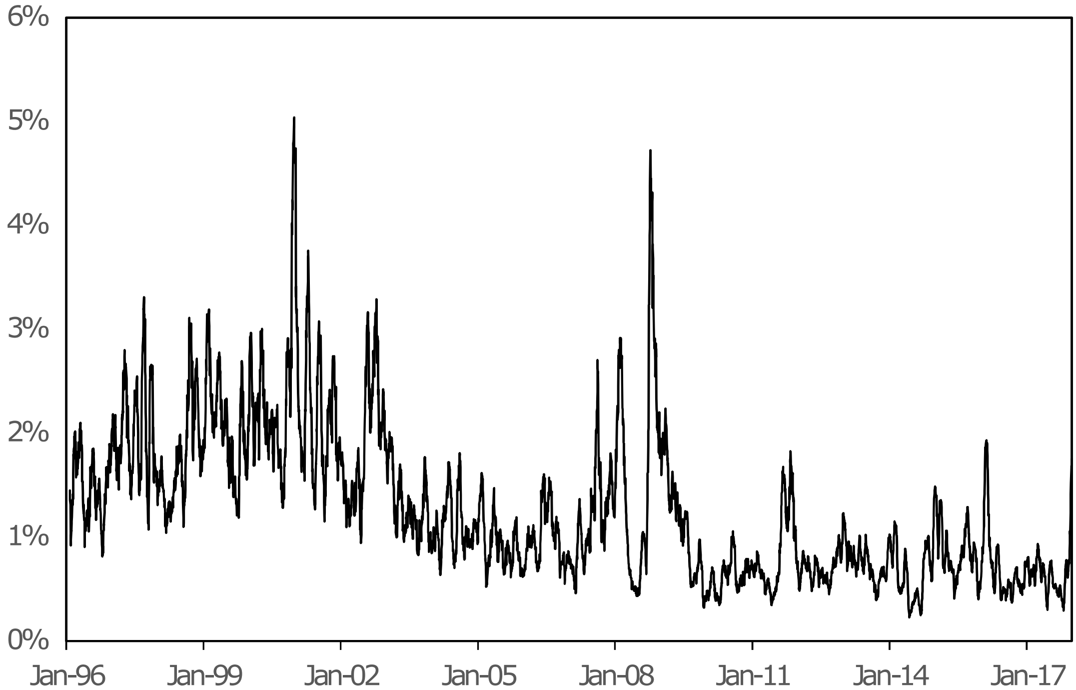


Figure 2. Horizontally extrapolated standard IVs in the RNM estimation

This Figure depicts the simulated call and put Black and Scholes (1973) implied volatility (BS-IV) curves, extracted from option prices across a continuum of strikes (dotted lines; considered to be the true unobserved IVs). It also shows the interpolated and extrapolated BS-IV curves, extracted from simulated option prices observed across discrete strikes (solid lines; considered to be the empirically observed IVs). We use Proposition 3.3 to simulate the call and put BS-IV curves, using the following parameters: the current underlying price is $S_t = 100$, the time-to-maturity is $\tau = 1/12$, the risk-free rate is $r_f = 3\%$, the volatility parameter is $\sigma = 30\%$, and the expected stock return is $r_S = r_f + 3\%$. For simplicity, we assume that the stock pays no dividends. The empirically observed BS-IVs are given by interpolating/extrapolating them across discrete moneyness K/S levels. We assume that options trade at $K = 94, 95, \ldots, 108$ (i.e., moneyness equals $0.94, 0.95, \ldots, 1.08$). We interpolate out-of-the-money call and put option BS-IVs, separately, by the cubic Hermite polynomial interpolation, and extrapolate horizontally, beyond the observed moneyness range. The level of moneyness, and the implied volatility (in percent) are shown on the x- and y-axis, respectively.



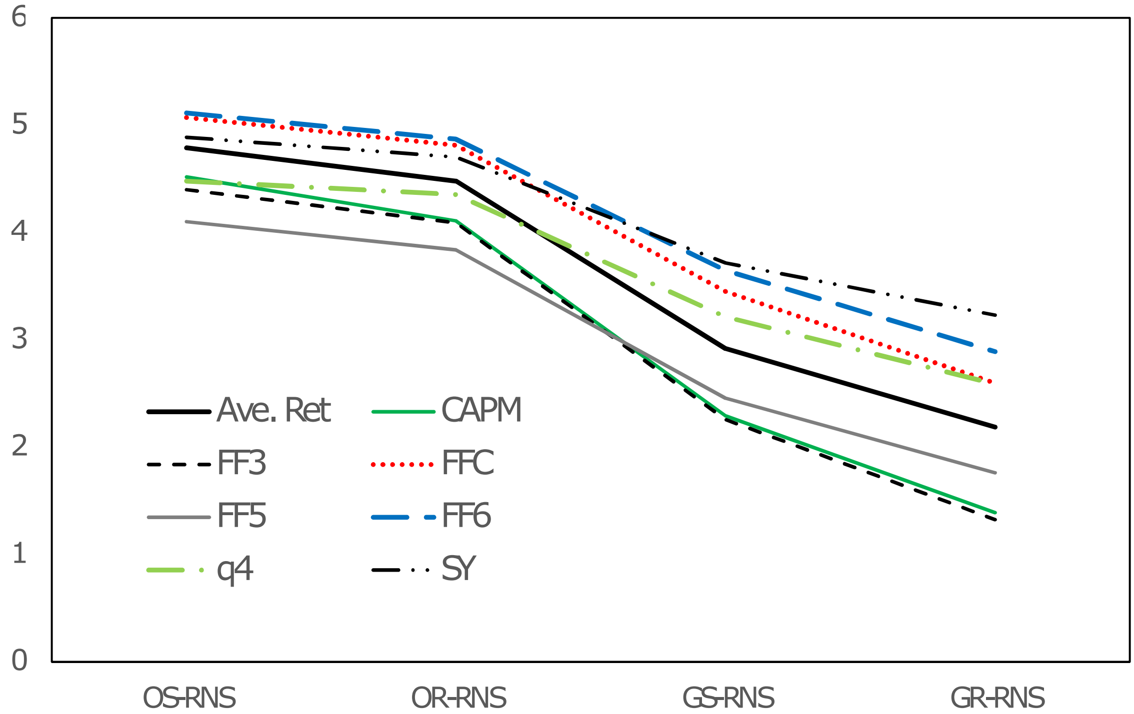
(a) Estimated daily 30-day SSD of S&P 500



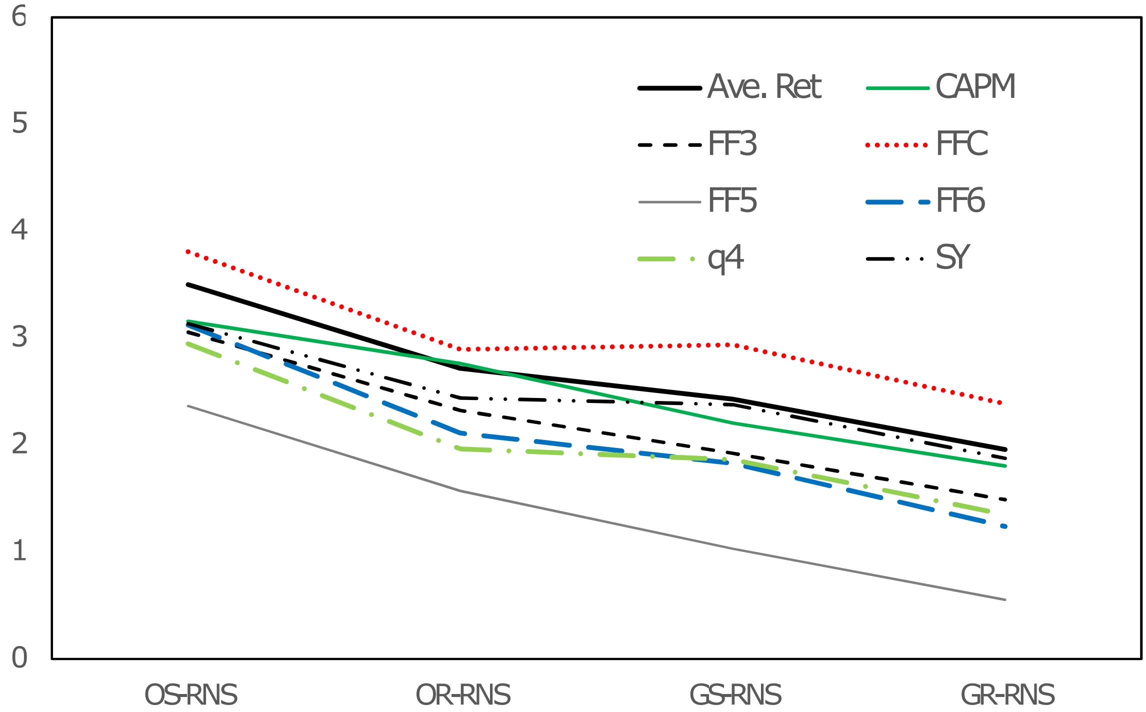
(b) Estimated daily 30-day SSD of S&P 500 (21-day moving average of the absolute value)

Figure 3. Estimated daily 30-day SSD of the S&P 500

Figure (a) depicts the daily 30-day SSD of the S&P 500 index estimated from the 30-day forward at-the-money call and put option prices recorded in the OptionMetrics Standardized Options file. Figure (b) depicts the 21-day moving average of the absolute value of the daily estimated 30-day SSD. The estimation period spans from the beginning of January 1996 to the end of December 2017. The unit of y-axis in the both Figures is the percentage value of SSD per year.



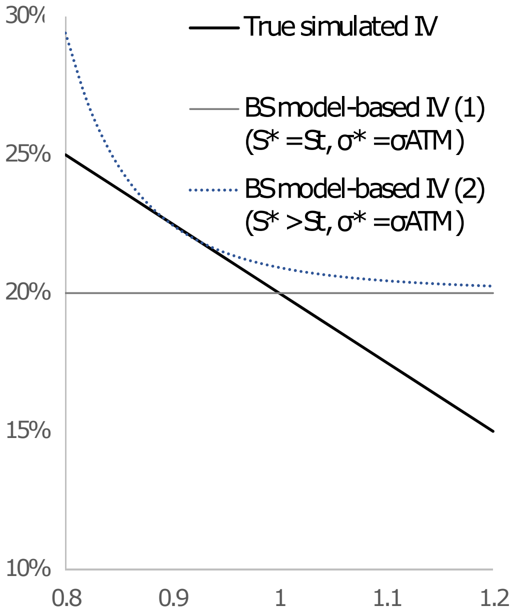
(a) Equally-weighted portfolios



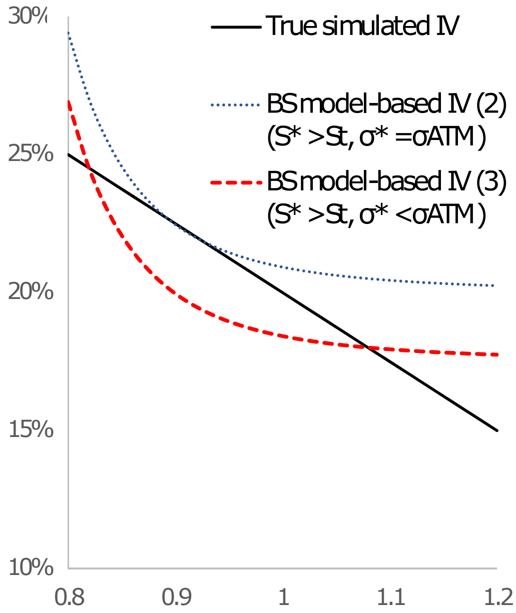
(b) Value-weighted portfolios

Figure 4. The t-statistics of the average returns and alphas of the RNS-sorted long-short spread portfolios based on the four alternative RNS estimation specifications

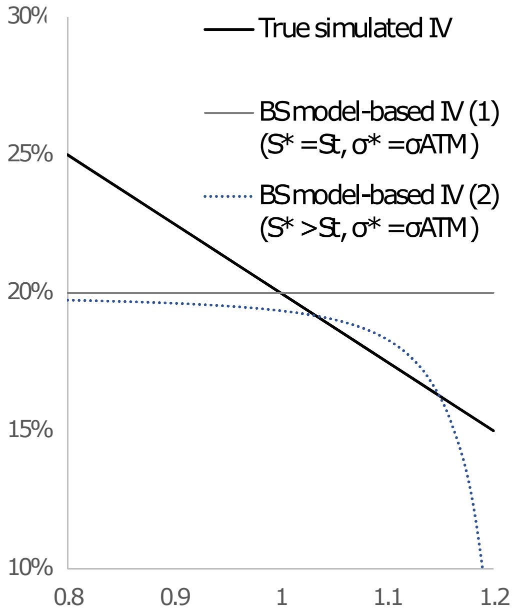
The x-axis stands for the estimation specification of the RNS. The first letter of the prefix denotes the formula used to estimate the RNS (either Original or Generalized). The second letter of the prefix denotes the IVs used for the inter-/extrapolation (either Standard IV or Robust IV). The y-axis stands for the value of the t-statistics.



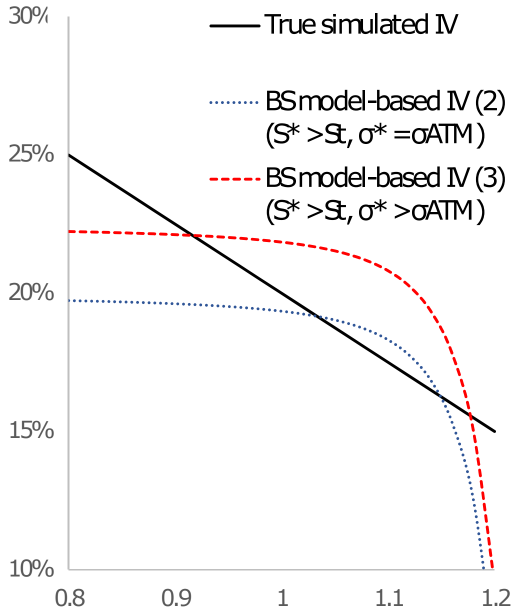
(a) Call IVs (1): the effect of a higher S_t^*



(b) Call IVs (2): the effect of a lower σ^*



(c) Put IVs (1): the effect of a higher S_t^*



(d) Put IVs (2): the effect of a higher σ^*

Figure 5. Implied stock price approach: Schematic graphs when the true BS-IV curve is negatively skewed

These four Figures illustrate how the BS model-based implied stock price approach chooses the implied stock price S_t^* and the BS volatility parameter σ^*. The black thick line in each Figure depicts the true simulated BS-IV curve as a function of moneyness $IV(K/S_t) = \sigma_{ATM} - (K/S_t - 1)/4$. The BS model-based implied stock price approach chooses S_t^* and σ^*, so that the BS-IV curve of the BS model-based option prices is the closest to the true simulated BS-IV curve. Figures (a) and (c) depict the effect of choosing a higher value for S_t^* (from the gray thin line to the blue dotted line), for the call option and put option cases, respectively. Figures (b) and (d) depict the effect of choosing a lower (higher) value of σ^* (from the blue dotted line to the red dashed line) for the call (put) option case, respectively. The x-axis is moneyness and the y-axis is the IV in percent.

Table 1. Effect of market frictions to RNMs: Simulation results

Entries report RNMs, based on simulated data, under four alternative estimation specifications. We generate simulated option prices, assuming that the stock price follows a log-normal distribution, equation (17). We set $S_t = 100$, $r_f = 3\%$, $\tau = 1/12$, and $\sigma = 30\%$. We consider three cases of the risk-neutral expected stock return r_S: $r_S = r_f$ (zero SSD case), $r_S = r_f + 3\%$ (positive SSD case), and $r_S = r_f - 3\%$ (negative SSD case). Under this setup, the true MFIV, RNS, and RNK are 30%, zero, and three, respectively, regardless of the value of r_S. The first column specifies the RNM estimation specifications. The first letter denotes the type of the formula ("Original" BKM formula, or the "Generalized" BKM formula). The second letter denotes the type of the interpolated-extrapolated IVs ("Standard" IVs, or the "Robust" IVs.)

	Zero SSD			Positive SSD			Negative SSD		
	MFIV	RNS	RNK	MFIV	RNS	RNK	MFIV	RNS	RNK
OS	30.0%	0.00	3.00	30.0%	0.11	3.00	30.0%	-0.11	3.00
OR	30.0%	0.00	3.00	30.0%	0.09	3.00	30.0%	-0.09	3.00
GS	30.0%	0.00	3.00	30.0%	0.02	2.99	30.0%	-0.02	3.01
GR	30.0%	0.00	3.00	30.0%	0.00	2.99	30.0%	0.00	3.00

Table 2. Summary statistics of SSD, and significance of mean absolute SSD

Entries in Panels A and B report the summary statistics of the SSD, and its absolute value calculated from daily and end-of-month SSD values, respectively. We calculate SSD for all available U.S. common stocks traded at NYSE/Amex/NASDAQ, for each trading day. P5 and P95 denote the fifth and 95-th percentiles of the pooled stock-day observations, respectively. The unit of SSD summary statistics is % per year. Entries in Panel C report the number of individual stocks, whose time-series mean absolute SSD is greater than zero, 1%, 2%, and 3%, respectively. A stock is included in this number, if the t-statistic is greater than three, where the null hypothesis is that the mean absolute SSD of the stock equals the given reference point. Entries in columns labeled "Daily" and "End-of-month" are computed using daily and monthly observations, respectively. We report the number and percentage, in squared brackets, of stocks, whose mean absolute SSD exceeds a reference value; we consider three groups of stocks consisting of at least 21 (12), 1200 (60), and 2500 (120) non-missing observations at a daily (monthly) frequency, respectively. The first row reports the number of stocks within each one of the groups. The estimation period spans January 1996 to December 2017.

Panel A: Summary statistics: Daily SSD						
	Obs	Mean	St. dev.	P5	Median	P95
SSD	12,469,967	-1.1	15.9	-17.0	-0.6	14.0
|SSD|	12,469,967	6.8	14.4	0.2	2.8	24.8
Panel B: Summary statistics: End-of-month SSD						
	Obs	Mean	St. dev.	P5	Median	P95
SSD	596,679	-0.9	16.2	-17.1	-0.4	14.4
|SSD|	596,679	7.0	14.7	0.2	2.8	25.2

Panel C: Number of stocks with significant mean absolute SSD						
	Daily			End-of-month		
	Valid observations			Valid observations		
	≥ 21	≥ 1200	≥ 2500	≥ 12	≥ 60	≥ 120
Number of stocks	6791	3357	1836	6219	3277	1833
mean(|SSD|) > 0	6692	3337	1827	5748	3177	1797
	[98.5]	[99.4]	[99.5]	[92.4]	[96.9]	[98.0]
mean(|SSD|) > 1%	6631	3308	1807	5290	2986	1689
	[97.6]	[98.5]	[98.4]	[85.1]	[91.1]	[92.1]
mean(|SSD|) > 2%	5968	2809	1390	3989	2139	1075
	[87.9]	[83.7]	[75.7]	[64.1]	[65.3]	[58.6]
mean(|SSD|) > 3%	5139	2265	1019	2899	1470	668
	[75.7]	[67.5]	[55.5]	[46.6]	[44.9]	[36.4]

Table 3. O-RNMs and G-RNMs: Summary Statistics

Entries in Panel A report the summary statistics of SSD, the three estimated risk-neutral moments (RNMs), model-free implied volatility (MFIV), risk-neutral skewness (RNS), and risk-neutral kurtosis (RNK), computed by the original and our generalized BKM formulae (prefix “O” and ”G”, respectively), separately. We also compute the difference between the estimated O-RNMs and the corresponding G-RNMs (with prefix “Δ”). We estimate these option-implied measures at the end of each month, from January 1996 to December 2017 (264 months). P5 and P95 denote the fifth and 95th percentile, respectively. The unit of SSD is % per 30-days, and that of MFIV is % (per year). Entries in Panel B report the Pearson and Spearman pairwise correlations between SSD and RNMs, SSD and ΔRNM, and O-RNM and G-RNM.

	SSD	O-MFIV	G-MFIV	O-RNS	G-RNS	O-RNK	G-RNK	ΔMFIV	ΔRNS	ΔRNK
Panel A: Summary statistics										
Mean	-0.07	52.11	51.99	-0.20	-0.19	3.64	3.62	0.12	-0.01	0.02
St. dev.	1.33	26.99	26.68	0.51	0.48	1.16	1.16	1.52	0.23	0.39
P5	-1.40	21.54	21.60	-0.87	-0.80	2.88	2.88	-0.47	-0.30	-0.14
Median	-0.03	45.74	45.71	-0.24	-0.21	3.28	3.26	-0.02	-0.01	0.01
P95	1.18	103.58	103.08	0.58	0.50	5.72	5.70	0.89	0.27	0.21
Obs	582,796	582,796	582,796	582,796	582,796	582,796	582,796	582,796	582,796	582,796
Panel B: Pairwise correlations										
		SSD and RNMs						SSD and ΔRNM		
		O-MFIV	G-MFIV	O-RNS	G-RNS	O-RNK	G-RNK	ΔMFIV	ΔRNS	ΔRNK
Pearson		-0.11	-0.07	0.36	-0.01	0.13	0.04	-0.72	0.82	0.29
Spearman		-0.08	-0.07	0.34	-0.01	0.07	0.09	-0.78	0.97	-0.11
		O-RNM and G-RNM								
		MFIV		RNS		RNK				
Pearson		1.00		0.90		0.94				
Spearman		1.00		0.88		0.96				

Table 4. Decile portfolio sort based on the estimated RNS

Entries report the average post-ranking returns of the equally- and value-weighted decide portfolios formed based on the estimated O-RNS and G-RNS (Panel A), as well as the average post-ranking returns, and the risk-adjusted returns (α) of the high-minus-low (10 minus 1) spread portfolios (Panel B). We estimate α's with respect to the CAPM, the Fama and French (1993) three-factor model (FF3), the Carhart (1997) four-factor model (FFC), the Fama and French (2018) five- and six-factor models (FF5 and FF6), the Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY). At the end of each month t, we sort stocks in ascending order based on the estimated O-RNS or G-RNS, and then we form equally- and value-weighted decile portfolios. Then, we calculate the post-ranking return of these portfolios, and the highest minus lowest (10 minus 1) spread portfolios in the succeeding month-($t+1$). The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses. The unit of the average returns and alphas is % per month. N in the last row of Panel A is the average number of stocks in each decile portfolio in each month.

	Equally-weighted				Value-weighted			
	O-RNS		G-RNS		O-RNS		G-RNS	
Panel A: Average return of the decile portfolios								
1 (lowest)	0.36	(1.12)	0.78	(2.60)	0.44	(1.58)	0.67	(2.54)
2	0.59	(1.69)	0.79	(2.28)	0.70	(2.50)	0.74	(2.65)
3	0.73	(1.93)	0.82	(2.23)	0.86	(2.98)	0.82	(2.75)
4	0.76	(1.92)	0.96	(2.40)	0.87	(2.74)	0.95	(3.14)
5	0.81	(1.97)	0.90	(2.23)	1.06	(3.38)	1.04	(3.37)
6	0.86	(2.05)	0.86	(1.98)	1.07	(3.35)	1.02	(2.93)
7	1.03	(2.33)	0.78	(1.75)	1.22	(3.71)	0.94	(2.65)
8	1.02	(2.25)	0.80	(1.69)	0.91	(2.56)	0.89	(2.48)
9	1.29	(2.67)	1.01	(2.04)	1.24	(3.43)	0.99	(2.43)
10 (highest)	1.50	(3.30)	1.25	(2.83)	1.23	(3.78)	1.07	(3.38)
N	220.8		220.8		220.8		220.8	
Panel B: Average returns and alphas of the spread portfolios								
Ave. Ret	1.14	(4.79)	0.48	(2.19)	0.79	(3.50)	0.40	(1.96)
α_{CAPM}	0.90	(4.52)	0.25	(1.39)	0.74	(3.16)	0.38	(1.80)
α_{FF3}	0.90	(4.40)	0.22	(1.32)	0.65	(3.06)	0.27	(1.49)
α_{FFC}	1.14	(5.07)	0.47	(2.60)	0.81	(3.81)	0.41	(2.39)
α_{FF5}	1.06	(4.10)	0.38	(1.76)	0.53	(2.36)	0.11	(0.55)
α_{FF6}	1.21	(5.11)	0.54	(2.89)	0.65	(3.12)	0.22	(1.24)
α_{q4}	1.37	(4.48)	0.65	(2.59)	0.73	(2.95)	0.29	(1.35)
α_{SY}	1.46	(4.89)	0.79	(3.23)	0.78	(3.14)	0.39	(1.88)

Table 5. Factor loadings obtained from regressions of the RNS-sorted value-weighted spread portfolios on factors

Entries report the factor loadings obtained from regressing the RNS-sorted value-weighted decile spread portfolios returns on factors. The factor models are the CAPM (MKT), Fama and French (1993) three-factor model (MKT, SMB, HML), Carhart (1997) four-factor model (MKT, SMB, HML, UMD), Fama and French (2018) five-factor model (MKT, SMB(FF5), HML, RMW, CMA) and six-factor model (MKT, SMB(FF5), HML, RMW, CMA, UMD), Hou et al. (2015) q-factor model (MKT(q4), ME(q4), IA(q4), ROE(q4)), and the Stambaugh and Yuan (2017) mispricing-factor model (MKT, SMB(SY), MGMT, PERF). Columns labeled "O" ("G") use O-RNS (G-RNS) as a sorting variable. The post-ranking return period spans February 1996 to December 2017 (263 months). *t*-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses.

	CAPM		FF3		FFC		FF5		FF6		q4		SY	
	O	G	O	G	O	G	O	G	O	G	O	G	O	G
MKT	0.09	0.04	0.10	0.05	0.00	-0.04	0.15	0.12	0.08	0.05			0.04	-0.03
	(1.30)	(0.55)	(1.58)	(0.85)	(0.04)	(-0.82)	(2.20)	(1.81)	(1.40)	(0.98)			(0.57)	(-0.43)
SMB			0.13	0.17	0.17	0.20								
			(1.30)	(1.47)	(2.15)	(2.19)								
HML			0.31	0.38	0.22	0.30	0.15	0.19	-0.02	0.04				
			(3.00)	(3.81)	(2.20)	(3.03)	(1.20)	(1.65)	(-0.19)	(0.35)				
UMD					-0.24	-0.22			-0.26	-0.24				
					(-3.91)	(-3.76)			(-4.41)	(-4.51)				
SMB(FF5)							0.19	0.25	0.24	0.29				
							(1.86)	(2.69)	(3.08)	(4.26)				
RMW							0.12	0.18	0.19	0.25				
							(0.77)	(1.20)	(1.67)	(1.99)				
CMA							0.23	0.23	0.32	0.32				
							(1.13)	(1.51)	(2.11)	(2.61)				
MKT(q4)											0.07	0.04		
											(1.11)	(0.66)		
ME(q4)											0.06	0.11		
											(0.53)	(0.74)		
IA(q4)											0.47	0.56		
											(2.92)	(3.89)		
ROE(q4)											-0.26	-0.21		
											(-1.88)	(-1.97)		
SMB(SY)													0.16	0.20
													(1.42)	(1.47)
MGMT													0.18	0.18
													(1.52)	(1.42)
PERF													-0.17	-0.17
													(-1.90)	(-1.98)
Alpha	0.74	0.38	0.65	0.27	0.81	0.41	0.53	0.11	0.65	0.22	0.73	0.29	0.78	0.39
	(3.16)	(1.80)	(3.06)	(1.49)	(3.81)	(2.39)	(2.36)	(0.55)	(3.12)	(1.24)	(2.95)	(1.35)	(3.14)	(1.88)
Adj. R^2	0.01	-0.01	0.08	0.12	0.19	0.22	0.10	0.15	0.23	0.27	0.11	0.14	0.07	0.08

Table 6. Robustness analysis: RNS in Fama-MacBeth regressions

Entries report the results from the Fama and MacBeth (1973) regression of the stock returns on the estimated RNS, firms' and stocks' characteristics, and SSD. Columns (1) to (4) (Columns (5) to (8)) report the results using O-RNS (G-RNS) as a regressor. At any given month, we conduct two alternative types of regressions for the cross-sectional regression: the ordinary least square (OLS), and the value-weighted least square (VWLS). The last row indicates the type of the cross-sectional regression. Beta is the market beta, SIZE is the log market capitalization, log(B/M) is the log book-to-market, Profit is the operational profit, Invest is the annual growth rate of the total asset, Momentum is the past 11 months cumulative return $R_{t-12,t-1}$, and Reversal is the previous month return $R_{t-1,t}$. Adj. R^2 is the time-series average of the adjusted R^2 of the cross-sectional regressions. N is the time-series average of the number of stocks in the cross-sectional regressions. The return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses.

FM regression of stock returns on RNS and characteristic variables								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
O-RNS	0.73	0.71	0.70	0.43				
	(3.94)	(3.61)	(7.15)	(3.16)				
G-RNS					0.39	0.49	0.36	0.19
					(1.86)	(2.02)	(3.96)	(1.29)
BETA			-0.07	-0.11			-0.07	-0.10
			(-0.25)	(-0.32)			(-0.24)	(-0.30)
SIZE			0.03	-0.05			0.01	-0.05
			(0.46)	(-0.89)			(0.15)	(-1.07)
log(B/M)			0.11	0.01			0.11	0.01
			(1.25)	(0.14)			(1.28)	(0.10)
Profit			0.13	0.08			0.13	0.08
			(2.85)	(1.92)			(2.86)	(1.89)
Invest			-0.05	-0.03			-0.05	-0.03
			(-1.44)	(-0.86)			(-1.48)	(-0.86)
Momentum			0.13	0.26			0.11	0.25
			(0.48)	(0.94)			(0.42)	(0.89)
Reversal			-0.90	-1.42			-1.04	-1.54
			(-1.63)	(-2.08)			(-1.89)	(-2.27)
Intercept	1.06	1.13	0.65	1.78	0.99	1.05	0.89	1.84
	(2.46)	(3.49)	(0.60)	(1.90)	(2.30)	(3.21)	(0.83)	(1.95)
Adj. R^2	0.25%	-3.34%	7.16%	3.07%	0.20%	-3.36%	7.11%	3.06%
N	2208	2208	2057	2057	2208	2208	2057	2057
Regress type	OLS	VWLS	OLS	VWLS	OLS	VWLS	OLS	VWLS

Table 7. Robustness analysis: Quintile portfolios sorted by the estimated RNS

Entries report the average post-ranking returns of the equally- and value-weighted quintile portfolios, formed based on the estimated O-RNS and G-RNS (Panel A). They also report the average post-ranking returns, and the risk-adjusted returns (α) of the high-minus-low (5 minus 1) spread portfolio (Panel B). We estimate α's with respect to the CAPM, Fama and French (1993) three-factor model (FF3), Carhart (1997) four-factor model (FFC), Fama and French (2018) five- and six-factor models (FF5 and FF6), Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY). At the end of each month t, we sort stocks in ascending order based on the estimated O-RNS or G-RNS, and we form equally- and value-weighted quintile portfolios. Then, we calculate the return of these portfolios, and that of the long-short (5 minus 1) spread portfolios in the succeeding month-($t+1$). The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses. The unit of the average returns and alphas is % per month.

	Equally-weighted				Value-weighted			
	O-RNS		G-RNS		O-RNS		G-RNS	
Panel A: Average return of the quintile portfolios								
1 (lowest)	0.48	(1.43)	0.78	(2.45)	0.57	(2.11)	0.71	(2.67)
2	0.74	(1.94)	0.89	(2.33)	0.86	(2.94)	0.87	(2.97)
3	0.83	(2.02)	0.88	(2.12)	1.07	(3.48)	1.03	(3.27)
4	1.03	(2.30)	0.79	(1.73)	1.08	(3.23)	0.90	(2.61)
5 (highest)	1.40	(3.00)	1.13	(2.43)	1.23	(3.66)	1.02	(2.87)
Panel B: Average returns and alphas of the spread portfolios								
Ave. Ret	0.92	(4.20)	0.35	(1.66)	0.66	(3.50)	0.31	(1.48)
α_{CAPM}	0.69	(3.75)	0.11	(0.68)	0.56	(3.08)	0.20	(1.01)
α_{FF3}	0.68	(3.81)	0.09	(0.57)	0.48	(2.98)	0.10	(0.62)
α_{FFC}	0.92	(4.63)	0.34	(2.04)	0.63	(3.98)	0.26	(1.60)
α_{FF5}	0.86	(3.67)	0.28	(1.26)	0.44	(2.37)	0.07	(0.38)
α_{FF6}	1.01	(4.82)	0.44	(2.41)	0.54	(3.32)	0.18	(1.06)
α_{q4}	1.15	(4.08)	0.54	(2.08)	0.66	(3.14)	0.23	(1.11)
α_{SY}	1.23	(4.48)	0.66	(2.73)	0.67	(3.56)	0.27	(1.41)

Table 8. Decile portfolios sorted by the dividend non-adjusted RNS

Entries report the average post-ranking returns of the equally- and value-weighted decide portfolios, formed based on the estimated dividend non-adjusted O-RNS and G-RNS (Panel A). They also report the average post-ranking returns and the risk-adjusted returns (α) of the high-minus-low (10 minus 1) spread portfolios (Panel B). The dividend non-adjusted RNS is estimated by setting $\widetilde{D}_{t,T} = 0$, in Proposition 3.1. We estimate α's with respect to the CAPM, Fama and French (1993) three-factor model (FF3), Carhart (1997) four-factor model (FFC), Fama and French (2018) five- and six-factor models (FF5 and FF6), Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY). At the end of each month t, we sort stocks in ascending order, based on the estimated O-RNS or G-RNS and we form equally- and value-weighted decile portfolios. Then, we calculate the return of these portfolios and that of the long-short (10 minus 1) spread portfolios in the succeeding month-($t + 1$). The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses. The unit of the average returns and alphas is % per month.

	Equally-weighted				Value-weighted			
	O-RNS		G-RNS		O-RNS		G-RNS	
Panel A: Average return of the decile portfolios								
1 (lowest)	0.43	(1.41)	0.75	(2.63)	0.53	(1.97)	0.74	(2.88)
2	0.57	(1.68)	0.81	(2.43)	0.71	(2.64)	0.75	(2.78)
3	0.71	(1.89)	0.90	(2.48)	0.82	(2.82)	0.84	(2.73)
4	0.78	(1.99)	0.92	(2.33)	0.88	(2.74)	0.94	(2.98)
5	0.82	(2.00)	0.87	(2.14)	1.06	(3.23)	1.07	(3.27)
6	0.83	(1.97)	0.83	(1.89)	1.05	(3.17)	0.89	(2.55)
7	0.97	(2.17)	0.81	(1.79)	1.21	(3.46)	1.12	(3.07)
8	1.05	(2.28)	0.84	(1.77)	0.97	(2.62)	0.89	(2.32)
9	1.27	(2.60)	0.99	(1.96)	1.17	(3.18)	0.90	(2.16)
10 (highest)	1.50	(3.23)	1.24	(2.72)	1.30	(4.00)	1.06	(3.25)
Panel B: Average returns and alphas of the spread portfolios								
Ave. Ret	1.06	(4.16)	0.49	(2.04)	0.77	(3.73)	0.32	(1.64)
α_{CAPM}	0.78	(3.72)	0.20	(1.04)	0.67	(3.06)	0.24	(1.24)
α_{FF3}	0.78	(3.72)	0.18	(1.09)	0.59	(3.11)	0.14	(0.83)
α_{FFC}	1.03	(4.56)	0.44	(2.47)	0.72	(3.83)	0.29	(1.87)
α_{FF5}	0.97	(3.79)	0.40	(1.87)	0.52	(2.63)	0.08	(0.45)
α_{FF6}	1.13	(4.80)	0.56	(3.07)	0.61	(3.36)	0.19	(1.18)
α_{q4}	1.29	(4.24)	0.66	(2.64)	0.70	(3.20)	0.22	(1.11)
α_{SY}	1.37	(4.61)	0.80	(3.29)	0.73	(3.38)	0.32	(1.68)

Table 9. Decile portfolio sort based on ΔRNS

Entries report the average post-ranking returns of the equally- and value-weighted decile portfolios formed based on the ΔRNS, which is the difference between O-RNS and G-RNS (Panel A), as well as the average post-ranking returns and the risk-adjusted returns (α) of the high-minus-low (10 minus 1) spread portfolios (Panel B). We estimate α's with respect to the CAPM, Fama and French (1993) three-factor model (FF3), Carhart (1997) four-factor model (FFC), Fama and French (2018) five- and six-factor models (FF5 and FF6), Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY). At the end of each month t, we sort stocks in ascending order, based on the estimated ΔRNS, and we form equally- and value-weighted decile portfolios. Then, we calculate the return of these portfolios and that of the long-short (10 minus 1) spread portfolios in the succeeding month-$(t+1)$. The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses. The unit of the average returns and alphas is % per month. N in the last row of Panel A is the average number of stocks in each decile portfolio.

	ΔRNS-sorted portfolios			
	Equally-weighted		Value-weighted	
Panel A: Average return of the decile portfolios				
1 (lowest)	-0.02	(-0.04)	0.22	(0.66)
2	0.44	(1.10)	0.59	(2.07)
3	0.62	(1.59)	0.65	(2.07)
4	0.80	(2.01)	0.73	(2.31)
5	0.92	(2.21)	0.82	(2.62)
6	0.92	(2.20)	1.01	(3.05)
7	1.01	(2.47)	1.02	(3.22)
8	1.19	(2.90)	0.90	(2.81)
9	1.40	(3.39)	1.14	(3.70)
10 (highest)	1.66	(3.84)	1.41	(4.36)
N	220.8		220.8	
Panel B: Average returns and alphas of the spread portfolios				
Ave. Ret	1.68	(9.70)	1.19	(5.69)
α_{CAPM}	1.59	(9.23)	1.11	(4.80)
α_{FF3}	1.63	(9.56)	1.14	(4.71)
α_{FFC}	1.71	(9.41)	1.27	(5.07)
α_{FF5}	1.71	(8.71)	1.12	(4.62)
α_{FF6}	1.76	(8.85)	1.21	(4.97)
α_{q4}	1.91	(8.56)	1.35	(4.84)
α_{SY}	1.82	(7.39)	1.33	(4.59)

Table 10. Dependent bivariate sort: First by $|SSD|$ and then by O-RNS

Entries report the result of the bivariate dependent sort, where we first sort stocks based on the absolute value of the estimated SSD, and then by O-RNS. At the end of each month, we first sort stocks into four subgroups based on the absolute value of the estimated SSD, and then within each absolute SSD group, we further sort stocks into quartile portfolios by the O-RNS criterion. Each column corresponds to the level of the first sorting variable, and each row reports the average return and alphas of the quartile spread portfolios with respect to the second sorting variable, O-RNS. The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses. The unit of the average returns and alphas is % per month.

	Equally-weighted				Value-weighted			
$\|SSD\|$	1 (low)	2	3	4 (high)	1 (low)	2	3	4 (high)
Highest O-RNS minus Lowest O-RNS spread portfolios								
Ave. Ret	0.26	0.37	0.57	1.51	0.36	0.18	0.63	1.15
	(1.43)	(1.94)	(2.67)	(6.18)	(1.85)	(0.78)	(2.93)	(4.58)
α_{CAPM}	0.07	0.17	0.39	1.28	0.29	0.05	0.56	1.06
	(0.45)	(1.00)	(2.02)	(5.88)	(1.48)	(0.22)	(2.85)	(4.17)
α_{FF3}	0.02	0.16	0.41	1.33	0.23	0.04	0.53	1.07
	(0.16)	(1.07)	(2.21)	(6.00)	(1.23)	(0.17)	(2.62)	(4.25)
α_{FFC}	0.19	0.31	0.66	1.53	0.31	0.09	0.68	1.24
	(1.36)	(1.88)	(3.44)	(6.19)	(1.68)	(0.44)	(3.44)	(4.36)
α_{FF5}	0.11	0.31	0.59	1.53	0.18	0.06	0.50	1.17
	(0.61)	(1.57)	(2.45)	(5.56)	(0.89)	(0.25)	(2.05)	(3.98)
α_{FF6}	0.22	0.40	0.75	1.65	0.24	0.09	0.60	1.27
	(1.42)	(2.22)	(3.71)	(6.11)	(1.21)	(0.43)	(2.81)	(4.16)
α_{q4}	0.29	0.45	0.86	1.84	0.26	0.18	0.75	1.42
	(1.59)	(2.03)	(3.02)	(6.06)	(1.25)	(0.81)	(3.09)	(4.24)
α_{SY}	0.36	0.56	1.02	1.85	0.27	0.17	0.71	1.38
	(2.10)	(2.80)	(3.75)	(5.73)	(1.33)	(0.75)	(3.04)	(3.73)

Table 11. SSD-adjusted regressions of the O-RNS-sorted portfolios

Entries report the intercepts of the regressions of SSD-adjusted excess returns of the value-weighted decide portfolios formed based on the estimated O-RNS, equation (33), on a set of risk factor(s). To this end, we consider the CAPM, Fama and French (1993) three-factor model (FF3), Carhart (1997) four-factor model (FFC), Fama and French (2018) five- and six-factor models (FF5 and FF6), Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY), separately. At the end of each month t, we sort stocks in ascending order based on the estimated O-RNS, and we form value-weighted decile portfolios. Then, we calculate the SSD-adjusted return of these portfolios and that of the spread portfolio in the succeeding month-$(t+1)$, and regress it on each set of risk-factors. The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses. The unit of SSD is % per 30 days and that of the alphas is % per month.

	O-RNS-sorted value-weighted decile portfolios										Spread
	1 (L)	2	3	4	5	6	7	8	9	10 (H)	10-1
SSD	-0.27	-0.11	-0.06	-0.04	-0.01	0.01	0.04	0.09	0.18	0.57	0.84
	(-16.54)	(-10.03)	(-7.84)	(-4.41)	(-1.35)	(0.74)	(5.02)	(8.77)	(11.99)	(9.86)	(12.54)
α_{CAPM}	-0.04	0.02	0.06	0.02	0.19	0.16	0.27	-0.11	0.14	-0.14	-0.10
	(-0.34)	(0.22)	(0.81)	(0.20)	(1.86)	(1.40)	(2.20)	(-0.81)	(0.85)	(-0.74)	(-0.44)
α_{FF3}	-0.06	0.02	0.06	0.03	0.23	0.16	0.28	-0.12	0.07	-0.25	-0.19
	(-0.51)	(0.22)	(0.83)	(0.31)	(2.41)	(1.61)	(2.31)	(-0.89)	(0.45)	(-1.46)	(-0.93)
α_{FFC}	-0.05	0.03	0.05	0.05	0.23	0.18	0.32	-0.04	0.22	-0.07	-0.02
	(-0.44)	(0.40)	(0.60)	(0.51)	(2.38)	(1.77)	(2.60)	(-0.25)	(1.49)	(-0.41)	(-0.09)
α_{FF5}	-0.01	0.00	0.04	0.02	0.24	0.17	0.29	-0.11	0.07	-0.31	-0.30
	(-0.09)	(0.01)	(0.55)	(0.16)	(2.37)	(1.65)	(2.32)	(-0.73)	(0.41)	(-1.59)	(-1.36)
α_{FF6}	-0.01	0.01	0.03	0.03	0.24	0.17	0.32	-0.06	0.17	-0.19	-0.18
	(-0.06)	(0.15)	(0.42)	(0.30)	(2.34)	(1.74)	(2.54)	(-0.37)	(1.13)	(-1.08)	(-0.88)
α_{q4}	-0.03	-0.04	0.03	0.06	0.29	0.24	0.38	0.07	0.24	-0.15	-0.11
	(-0.27)	(-0.54)	(0.37)	(0.50)	(2.68)	(2.08)	(2.97)	(0.48)	(1.26)	(-0.65)	(-0.46)
α_{SY}	0.01	-0.01	0.03	0.05	0.31	0.23	0.38	0.04	0.27	-0.03	-0.04
	(0.09)	(-0.18)	(0.35)	(0.43)	(2.92)	(1.97)	(2.97)	(0.24)	(1.62)	(-0.14)	(-0.16)

Table 12. Subsample analysis: Traded versus non-traded options

Entries report the average post-ranking returns, SSD, the average firm size (log market capitalization), relative bid-ask spread (BAS), and Amihud's (2002) illiquidity measure (Illiq) of the value-weighted decide portfolios, formed based on the estimated O-RNS (Panel A), as well as the average post-ranking returns and the risk-adjusted returns (α) of the high-minus-low (10 minus 1) spread portfolios (Panel B). We split our stock universe into two subgroups, based on the aggregate option trading volume, on each sorting date. The "traded" subgroup contains stocks, whose options have a non-zero trading volume. The "non-traded" subgroup contains stocks, whose options have zero trading volume. We estimate α's with respect to the CAPM, Fama and French (1993) three-factor model (FF3), Carhart (1997) four-factor model (FFC), Fama and French (2018) five- and six-factor models (FF5 and FF6), Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY), separately. For any given option trading volume group, at the end of each month t, we sort stocks in ascending order, based on the estimated O-RNS, and we form value-weighted decile portfolios. Then, we calculate the return of these portfolios, and the long-short (10 minus 1) spread portfolios in the succeeding month-$(t+1)$. We also calculate the value-weighted average of stock characteristics of the stocks in each decile portfolio. The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses. The unit of the average returns and alphas is % per month. N in the last row of Panel A is the average number of stocks in each decile portfolio.

	Stocks with non-zero option trading volume						Stocks with zero option trading volume					
	Return		SSD	SIZE	BAS	Iliq	Return		SSD	SIZE	BAS	Iliq
Panel A: Average return and characteristics of the decile portfolios												
1 (Low)	0.43	(1.56)	-0.24	17.2	0.38	0.08	0.68	(2.48)	-0.65	14.6	0.48	1.07
2	0.72	(2.60)	-0.10	17.5	0.34	0.07	0.96	(2.92)	-0.29	14.5	0.47	0.67
3	0.85	(2.90)	-0.06	17.4	0.33	0.07	0.84	(2.57)	-0.17	14.4	0.48	0.79
4	0.92	(2.95)	-0.03	17.3	0.33	0.09	1.11	(3.40)	-0.12	14.4	0.47	0.85
5	1.01	(3.12)	-0.01	17.1	0.33	0.09	1.03	(3.18)	-0.06	14.3	0.48	0.93
6	1.06	(3.34)	0.00	16.9	0.35	0.15	1.21	(3.38)	0.02	14.2	0.50	1.04
7	1.13	(3.43)	0.03	16.8	0.36	0.14	1.40	(4.05)	0.08	14.1	0.52	1.12
8	1.02	(2.90)	0.07	16.5	0.39	0.23	1.36	(3.57)	0.20	14.1	0.55	1.58
9	1.23	(3.42)	0.14	16.1	0.42	0.75	1.59	(4.20)	0.40	14.1	0.56	1.39
10 (Hi)	1.14	(3.19)	0.43	15.6	0.47	0.70	1.48	(4.35)	1.04	14.1	0.56	1.37
N	174.3						46.7					
Panel B: Average returns and alphas of the spread portfolios												
Ave. Ret	0.71	(2.91)					0.80	(3.82)				
α_{CAPM}	0.61	(2.65)					0.73	(3.62)				
α_{FF3}	0.51	(2.38)					0.68	(3.31)				
α_{FFC}	0.68	(3.04)					0.86	(3.75)				
α_{FF5}	0.43	(1.78)					0.64	(3.32)				
α_{FF6}	0.55	(2.41)					0.76	(4.00)				
α_{q4}	0.64	(2.35)					0.88	(3.70)				
α_{SY}	0.70	(2.81)					0.89	(3.27)				

Table 13. Summary statistics of the estimated IV skew measures

Entries in Panel A show the summary statistics of SSD, the XZZ IV slope measures calculated based on the standard IV (XZZ^o), and the robust IV (XZZ^r), separately, as well as the difference between the two XZZ measures. We estimate these option-implied measures at the end of each month from January 1996 to December 2017 (264 months). P5 and P95 stand for the fifth and 95th percentile, respectively. The unit of SSD is % per 30-day. The unit for other columns is the volatility in percentage. Entries in Panel B report the Pearson and Spearman pairwise correlations between SSD and the XZZ slope measures.

	SSD	XZZ^o	XZZ^r	ΔXZZ
Panel A: Summary statistics				
Mean	-0.07	6.85	6.33	0.53
St. dev.	1.32	14.78	11.24	8.98
P5	-1.40	-6.73	-2.49	-8.10
Median	-0.03	4.96	4.21	0.26
P95	1.18	27.28	23.34	9.60
Obs	582,686	582,686	582,686	582,686
Panel B: Pairwise correlations				
		SSD and XZZ measures		
		XZZ^o	XZZ^r	ΔXZZ
Pearson		-0.66	-0.07	-1.00
Spearman		-0.51	-0.05	-0.99
		XZZ^o and XZZ^r		
Pearson		0.80		
Spearman		0.80		

Table 14. Decile portfolio sort based on the XZZ IV slope measures

Entries report the average post-ranking returns of the equally- and value-weighted decide portfolios formed based on the estimated XZZ^o and XZZ^r (Panel A), as well as the average post-ranking returns, and the risk-adjusted returns (α) of the high-minus-low (10 minus 1) spread portfolios (Panel B). We estimate α's with respect to the CAPM, Fama and French (1993) three-factor model (FF3), the Carhart (1997) four-factor model (FFC), the Fama and French (2018) five- and six-factor models (FF5 and FF6), Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY). At the end of each month t, we sort stocks in ascending order based on the estimated XZZ^o or XZZ^r, and we form equally- and value-weighted decile portfolios. Then, we calculate the post-ranking return of each one of these portfolios, and of the long-short (10 minus 1) spread portfolios, in the succeeding month-($t+1$). The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation are reported in parentheses. The unit of the average returns and alphas is % per month. N in the last row of Panel A is the average number of stocks in each decile portfolio in each month.

	Equally-weighted				Value-weighted			
	XZZ^o		XZZ^r		XZZ^o		XZZ^r	
Panel A: Average return of the decile portfolios								
1 (lowest)	1.54	(2.88)	1.15	(2.17)	1.68	(4.09)	1.03	(2.78)
2	1.36	(3.26)	1.08	(2.40)	1.20	(4.20)	1.25	(4.23)
3	1.13	(3.13)	0.87	(2.36)	1.06	(4.09)	0.92	(3.63)
4	1.01	(2.86)	0.90	(2.47)	0.96	(3.67)	0.81	(3.15)
5	0.91	(2.57)	0.91	(2.52)	0.75	(2.81)	0.82	(3.04)
6	0.87	(2.29)	0.83	(2.21)	0.68	(2.25)	0.80	(2.68)
7	0.66	(1.68)	0.79	(2.03)	0.69	(2.21)	0.66	(2.19)
8	0.67	(1.58)	0.80	(1.97)	0.55	(1.50)	0.87	(2.59)
9	0.62	(1.45)	0.82	(1.91)	0.62	(1.56)	0.79	(2.04)
10 (highest)	0.19	(0.43)	0.80	(1.91)	0.28	(0.61)	0.65	(1.55)
N	220.7		220.7		220.7		220.7	
Panel B: Average returns and alphas of the spread portfolios								
Ave. Ret	-1.34	(-6.27)	-0.35	(-1.68)	-1.39	(-4.40)	-0.38	(-1.47)
α_{CAPM}	-1.19	(-5.69)	-0.17	(-0.88)	-1.44	(-4.35)	-0.44	(-1.87)
α_{FF3}	-1.25	(-6.30)	-0.22	(-1.15)	-1.47	(-4.49)	-0.44	(-1.90)
α_{FFC}	-1.36	(-5.93)	-0.36	(-1.72)	-1.50	(-4.22)	-0.42	(-1.70)
α_{FF5}	-1.35	(-5.80)	-0.40	(-1.84)	-1.26	(-3.93)	-0.23	(-1.06)
α_{FF6}	-1.42	(-5.90)	-0.49	(-2.22)	-1.30	(-3.82)	-0.23	(-0.98)
α_{q4}	-1.65	(-6.11)	-0.62	(-2.54)	-1.57	(-4.06)	-0.36	(-1.46)
α_{SY}	-1.55	(-5.35)	-0.59	(-2.24)	-1.36	(-3.71)	-0.17	(-0.67)

Table 15. SSD-adjusted regression of the XZZ-sorted portfolios

Entries report the intercepts of the regressions of SSD-adjusted excess returns of the value-weighted decide portfolios formed based on the estimated XZZ^o, equation (33), on a set of risk factor(s) of the CAPM, Fama and French (1993) three-factor model (FF3), Carhart (1997) four-factor model (FFC), Fama and French (2018) five- and six-factor models (FF5 and FF6), Hou et al. (2015) q-factor model (q4), and the Stambaugh and Yuan (2017) mispricing factor model (SY), respectively. At the end of each month t, we sort stocks in ascending order, based on the estimated XZZ^o, and we form value-weighted decile portfolios. Then, we calculate the SSD-adjusted return for each one of these portfolios, and the long-short (10 minus 1) spread portfolio, in the succeeding month-$(t+1)$, and regress it on any given set of risk-factors. The post-ranking return period spans February 1996 to December 2017 (263 months). t-statistics, adjusted for heteroscedasticity and autocorrelation, are reported in parentheses. The unit of SSD is % per 30 days and that of the alphas is % per month.

	O-RNS sorted value-weighted decile portfolios										Spread
	1 (lowest)	2	3	4	5	6	7	8	9	10 (highest)	10-1
SSD	1.09	0.18	0.06	0.00	-0.04	-0.08	-0.13	-0.21	-0.30	-0.71	-1.79
	(11.62)	(8.06)	(6.18)	(0.39)	(-7.58)	(-14.10)	(-17.25)	(-18.87)	(-17.47)	(-13.99)	(-12.97)
α_{CAPM}	-0.36	0.21	0.23	0.20	0.02	-0.07	-0.04	-0.19	-0.08	-0.02	0.35
	(-1.58)	(1.77)	(2.46)	(1.88)	(0.20)	(-0.78)	(-0.42)	(-1.67)	(-0.51)	(-0.08)	(1.16)
α_{FF3}	-0.42	0.20	0.23	0.20	0.01	-0.09	-0.05	-0.22	-0.15	-0.09	0.32
	(-1.77)	(1.65)	(2.38)	(1.93)	(0.08)	(-1.08)	(-0.52)	(-1.82)	(-1.02)	(-0.41)	(1.04)
α_{FFC}	-0.16	0.23	0.19	0.13	-0.03	-0.10	-0.06	-0.18	-0.04	0.12	0.28
	(-0.62)	(1.84)	(1.94)	(1.28)	(-0.30)	(-1.13)	(-0.57)	(-1.52)	(-0.28)	(0.62)	(0.78)
α_{FF5}	-0.31	0.12	0.17	0.03	-0.06	-0.12	-0.04	-0.07	0.02	0.19	0.50
	(-1.14)	(0.92)	(1.91)	(0.25)	(-0.69)	(-1.39)	(-0.34)	(-0.59)	(0.15)	(0.91)	(1.50)
α_{FF6}	-0.14	0.14	0.15	-0.01	-0.08	-0.12	-0.04	-0.05	0.08	0.32	0.46
	(-0.55)	(1.12)	(1.55)	(-0.12)	(-0.91)	(-1.40)	(-0.38)	(-0.46)	(0.55)	(1.67)	(1.28)
α_{q4}	0.02	0.21	0.16	0.07	-0.06	-0.12	-0.04	-0.09	0.06	0.25	0.23
	(0.05)	(1.41)	(1.74)	(0.60)	(-0.66)	(-1.30)	(-0.33)	(-0.71)	(0.33)	(1.05)	(0.61)
α_{SY}	0.08	0.23	0.14	-0.07	-0.10	-0.11	-0.02	-0.06	0.16	0.45	0.37
	(0.28)	(1.65)	(1.38)	(-0.80)	(-1.24)	(-1.03)	(-0.21)	(-0.50)	(0.94)	(2.23)	(0.99)

Table 16. Limitations of the implied stock price approach: Simulation results

Entries report the simulation results of the implied stock price approach. We use the Black and Scholes (1973) model to calculate the implied stock price. We simulate call and put option prices, assuming $S_t = 100$, $\tau = 1/6$, $r_f = 4\%$, $\widetilde{D}_{t,T} = 0.5$ for strikes $K = 80, 85, \ldots, 120$. The true implied volatility (IV) curve is modelled as a function of moneyness, $IV(K/S_t) = \sigma_{ATM} + k(K/S_t - 1)$ with $\sigma_{ATM} = 20\%$. We consider five alternative values for k; $k = -1/2, -1/4, 0, 1/4, 1/2$. Given these assumptions, we simulate a set of option prices using the Black and Scholes (1973) (BS) function with deterministic dividend payments. Then, we estimate the two parameters S_t^* and σ^* by minimizing the sum of squared errors between the simulated option prices and the theoretical option prices, based on the BS function $BS(S_t^*, K, T, r, q, \sigma^*, \widetilde{D}_{t,T})$. We use a set of call options and a set of put options, separately, as the observed option prices. Columns under Call (Put) options, report the estimation results, respectively. Given the estimate of S_t^*, we calculate the corresponding annualized estimated return wedge, $\widehat{\omega_{t,T}} = \Delta_t e^{r_f \tau}/\tau$, which would prevail if the assumption of the implied stock approach (equation (B.16)) holds.

	Call options			Put options		
k	S_t^*	$\widehat{\omega_{t,T}}$	σ^*	S_t^*	$\widehat{\omega_{t,T}}$	σ^*
-1/2	100.38	2.3%	17.9%	100.35	2.1%	21.1%
-1/4	100.17	1.0%	19.0%	100.21	1.2%	20.6%
0	100	0.0%	20.0%	100	0.0%	20.0%
1/4	99.86	-0.9%	21.1%	99.71	-1.7%	19.1%
1/2	99.72	-1.7%	22.2%	99.35	-4.0%	18.0%